ESSAY ON INEQUALITY, BUSINESS CYCLES, AND MACROECONOMIC POLICY

A Dissertation

by

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Submitted to the Office of Graduate and Professional Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

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May 2019

Major Subject: Economics

ABSTRACT

This dissertation investigates inequality, business cycles, and macroeconomic policies.

First, I investigate the quantitative implications of real wage rigidities and heterogeneity for two long-lasting puzzles in the business cycles literature: the low correlation between total hours worked and labor productivity and the large volatility of the labor wedge, defined as a gap between the marginal rate of substitution of aggregate leisure for aggregate consumption and the marginal product of aggregate labor. I shed light on these issues by extending a heterogeneous-agent model with an indivisible labor supply choice to real wage rigidities. I find that a small amount of real wage stickiness would be sufficient to resolve both anomalies when long-term wage contracts and heterogeneity are taken into account.

Second, I study the heterogeneous responses of consumption between the poor and the rich to government spending shocks. Government spending shocks have substantially different effects on consumers across the income distribution: consumption increases for the poor whereas it decreases for the rich in response to a rise in government expenditure. I shed light on this issue by incorporating a progressive tax scheme and productive public expenditure into a heterogeneous agent model economy with indivisible labor. The model economy is able to successfully match aggregate and disaggregate effects of government spending shocks on consumption. When the government increases its spending and accompanies it by a rise in tax progressivity, the poor are employed and increase their consumption since after-tax wage rates increase while the rich decrease their consumption because of a fall in after-tax wage rates.

Third, I also investigate the relation between monetary policy and inequality by asking how one affects the other: the effect of monetary policy on inequality and the impact of the long-run level of inequality on the effectiveness of monetary policy. To this end, I incorporate nominal wage contracts and cash-in-advance constraints into a heterogeneous agent model economy with indivisible labor. I find that expansionary monetary policy reduces income, wealth, and consumption inequalities mainly due to a rise in employment from the bottom of the distributions. There are heterogeneous effects on income across the wealth distribution: in response to an unanticipated monetary easing, households in the bottom of the wealth distribution benefit from an increase in employment while rich households benefit from a rise in the real asset returns in a relative sense. An unexpected monetary expansion also has asymmetric responses of consumption between the poor and the rich: asset-poor households increase their consumption while it falls for wealthy households. This implies that inflation hurts the rich more. I also find that the long-run prevailing levels of inequality matter for the effectiveness of monetary policy by determining the size of labor supply elasticity, which represents the shape of reservation wage distribution. All else being equal, a more equal economy is associated with more effective monetary policy in terms of output. I also provide empirical evidence for this model result using state-level panel data: the effects of monetary policy shocks on output are larger for low-inequality states.

DEDICATION

To my family, my teachers, and my friends

ACKNOWLEDGMENTS

I am deeply grateful to Dr. Sarah Zubairy. To the finishing line to my doctoral degree, she ran together with me. Whenever I met her, she had the best advice, which always led me to break through in the research. I also would like to express my special appreciation to my committee members. I would like to thank Dr. Dennis Jansen, Dr. Hwagyun Kim, and Dr. Yuzhe Zhang for their invaluable comments and suggestions. It was very fortunate for me to have advice from them. I also gratefully acknowledge Dr. Noh-Sun Kwark. His mentorship has enabled me to continue my research.

Special thanks are reserved for my family. Words cannot express how grateful I am to my parents, Seonsuk Ma and Yeongsuk Kim. They are my best teachers. My brother and sister, Hyunsang and Eunmi, also made me feel easy and encouraged my progress. I also would like to express my special appreciation to my wife, Wonhee Yun. Without her help, I could not have started the doctoral program. She made me always comfortable and free of any worries. With continuous support and encouragement from her, I was always reassured. My daughter, Hayeon Ma, is one of the greatest contributors to this dissertation. Her staying with me itself has meant a lot to me.

At the end I want to thank my Lord and Savior, Jesus Christ for everything.

CONTRIBUTORS AND FUNDING SOURCES

Contributors

This work was supported by an advisory committee consisting of Professors Sarah Zubairy, Dennis Jansen, and Yuzhe Zhang of the Department of Economics, and Hwagyun Kim of the Department of Finance.

All work for the dissertation was completed by the student independently. Chapter 3 is forthcoming at the Journal of Money, Credit and Banking.

Funding Sources

Graduate study was supported by a fellowship from Texas A&M University.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

1. INTRODUCTION

The unequal distributions of income and wealth have become a primary concern for economists and policy makers. Severe inequality is relevant to the implications of household-level heterogeneity on the dynamics of macroeconomic aggregates. In Chapter 2, I study the quantitative implications of real wage rigidities and heterogeneity for two long-lasting puzzles in the business cycles literature: the low correlation between total hours worked and labor productivity and the large volatility of the labor wedge, defined as a gap between the marginal rate of substitution of aggregate leisure for aggregate consumption and the marginal product of aggregate labor. I shed light on these issues by extending a heterogeneous-agent model with an indivisible labor supply choice to real wage rigidities. I find that a small amount of real wage stickiness would be sufficient to resolve both anomalies when long-term wage contracts and heterogeneity are taken into account.

In recent years, such inequality is particularly relevant for the stabilization role of economic policies since policies might have disparate effects across different segments of the population. In Chapter 3, I try to explain why government expenditure shocks affect consumers differently using a quantitative analysis based on a heterogeneous agent economy. By using the Consumer and Expenditure Survey (CEX), I empirically find that government spending shocks have substantially different impacts on different consumers: consumption increases for the poor whereas it decreases for the rich in response to a rise in government expenditure. In order to study the distributional effects of government spending shocks, I incorporate a progressive tax scheme and productive public expenditure into a heterogeneous agent model economy with indivisible labor. The model economy is able to successfully match aggregate and disaggregate effects of government spending shocks on consumption. When the government increases its spending and accompanies it by a rise in tax progressivity, the poor are employed and increase their consumption since after-tax wage rates increase while the rich decrease their consumption because of a fall in after-tax wage rates.

In Chapter 4, I investigate the relation between monetary policy and inequality by asking how one affects the other: the effect of monetary policy on inequality and the impact of the long-run level of inequality in the effectiveness of monetary policy. To this end, I incorporate nominal wage contracts and cash-in-advance constraints into a heterogeneous agent model economy with indivisible labor. I find that expansionary monetary policy reduces income, wealth, and consumption inequalities mainly due to a rise in employment from the bottom of the distributions. There are heterogeneous effects on income across the wealth distribution: in response to an unanticipated monetary easing, households in the bottom of the wealth distribution benefit from an increase in employment while rich households benefit from a rise in the real asset returns in a relative sense. An unexpected monetary expansion also has asymmetric responses of consumption between the poor and the rich: asset-poor households increase their consumption while it falls for wealthy households. This implies that inflation hurts the rich more. I also find that the long-run prevailing levels of inequality matter for the effectiveness of monetary policy by determining the size of of labor supply elasticity, which represents the shape of reservation wage distribution. All else being equal, a more equal economy is associated with more effective monetary policy in terms of output. I also provide empirical evidence for this model result using state-level panel data: the effects of monetary policy shocks on output are larger for low-inequality states.

2. REAL WAGE RIGIDITY, HETEROGENEITY, AND THE BUSINESS CYCLES

2.1 Introduction

Understanding labor market dynamics is important in the business cycle literature since labor income is a main source of total income; hence, fluctuations in variables related to labor market may directly affect economic welfare. Although the equilibrium business cycle models based on the representative agent have had a lot of success in accounting for most of the stylized facts of business cycles, they cannot account for some important issues regarding labor market dynamics. For example, the high volatility of the labor wedge, defined as a gap between the marginal rate of substitution (MRS) between consumption and leisure and the aggregate productivity (or marginal product of labor, MPL), is a well-known puzzle in the business cycle literature. In addition, the low correlation between aggregate hours of work and aggregate labor productivity is still an open question. These two puzzles are central issues in the literature on the real business cycle theory as they are closely related to welfare costs of the business fluctuations (Gali, Gertler and LopezSalido, 2007; Chang and Kim, 2007).[1] This study addresses both anomalies simultaneously by introducing heterogeneity[2] and real wage rigidities, which have often been abstracted in the neoclassical frameworks.[3] Once heterogeneity is taken into account, aggregate variables are not determined by the representative household's optimality condition, and they are nothing but the sum of individual variables. In this sense, heterogeneity across economic agents connotes aggregation bias, and analyzing behaviors of agents at the individual level is crucial to understand dynamics of the aggregate variables. Chang and Kim (2007), for instance, find that aggregation errors induced by heterogeneity of individual households endogenously generate

[1] Gali, Gertler and LopezSalido (2007) show that the wedge can be used as a measure of the lost surplus in the labor market, and Chang and Kim (2007) argue that the labor wedge may arise because of the low correlation between hours worked and productivity.

[2] In this paper, a term "heterogeneity" involves asset market incompleteness.

[3] Cho and Cooley (1995) incorporate nominal wage contracts into a real business cycle model, while Krusell and Smith (1998) among others build a heterogeneous-agent model with aggregate productivity shocks in the context of the neoclassical economies.

the labor wedge. Besides heterogeneity, according to Gali, Gertler and LopezSalido (2007) and Shimer (2009), labor market frictions including real wage rigidities are the key factors for labor market dynamics. However, most of the existing studies on wage stickiness are based on the representative agent (Cho and Cooley, 1995; Cho, Cooley and Phaneuf, 1997; Gali, Gertler and LopezSalido, 2007; Uhlig, 2007; Abbritti and Weber, 2010), and there have been surprisingly few attempts to analyze the implications of real wage rigidities for the dynamics of labor markets in the context of heterogeneous-agent economies.[4] Therefore, this study explores the quantitative relevance of wage rigidities and heterogeneity for labor market dynamics focusing on the two enduring anomalies.

To this end, I extend a heterogeneous-agent model with an indivisible labor supply choice to real wage rigidities. There are three main features in the model economy. First, a household is assumed to not fully insure against idiosyncratic productivity shocks that she faces: asset markets are incomplete as in Huggett (1993) and Aiyagari (1994). This feature generates rich heterogeneity across households' individual characteristics, such as employment, wealth, income, and consumption. Second, it is assumed that a labor supply decision for a household is indivisible, following Hansen (1985) and Chang and Kim (2007). It is well-known that extensive margins of hours worked are important to explain the variation in total hours of work. Third, the model economy allows for the presence of real wage rigidity following Gali, Gertler and LopezSalido (2007) and Shimer (2009), who suggest that labor market frictions including wage rigidities are important explaining the labor wedge. I assume that wage rigidities arise from wage contracts agreed to by households and firms as in the work of Cho and Cooley (1995) and Cho, Cooley and Phaneuf (1997).

The main findings of this study are summarized as follows. First, I find that the correlation coefficient between hours and productivity decreases when the index of wage stickiness or the length of wage contracts increases. This is because hours and productivity no longer

[4] Benigno and Ricci (2011) investigate the macroeconomic implications of downward nominal wage rigidities using a model economy with aggregate and idiosyncratic shocks. Schulz (2015) builds a search and matching model with heterogeneity and sorting, which endogenously generates wage rigidity.

move along the labor supply curve due to the wage contract. Heterogeneity also helps explain the low correlation between two macro variables, since the labor supply curve fluctuates in response to aggregate shocks (Chang and Kim, 2007). Second, I also find that the volatility of labor wedge increases with the index of wage rigidity or the wage contract length. More volatile hours worked and the lower correlation between aggregate hours and productivity, which are induced by wage rigidity, produce the larger cyclical movement in the labor wedge.[5] Heterogeneity also plays a role in accounting for the large volatile wedge since the wedge is endogenously generated when heterogeneity is taken into account. From these findings, I conclude that a small amount of real wage rigidity would be enough to resolve both puzzles when long-term wage contracts and heterogeneity are considered.

A large body of literature has provided various explanations for the low correlation between hours and productivity and the large volatile wedge. Key contributions are Benhabib, Rogerson and Wright (1991), Cho and Cooley (1995), Hall (1997), Gali, Gertler and LopezSalido (2007) and Shimer (2009). Benhabib, Rogerson and Wright (1991) and Hall (1997) consider exogenous shocks to the labor supply schedule for an explanation for the puzzles. Benhabib, Rogerson and Wright (1991) incorporate home production technology into a standard real business cycle framework, while Hall (1997) analyzes preference shifts as an exogenous shock to the labor supply curve. Cho and Cooley (1995) incorporate nominal wage contracts and monetary shocks into a real business cycle (RBC) model, and their model can generate the low correlation between hours and productivity. Other important works on the labor wedge are Gali, Gertler and LopezSalido (2007) and Shimer (2009). Gali, Gertler and LopezSalido (2007) study the efficiency gap (the labor wedge) as a measure of the welfare costs in the labor market based on a New Keynesian model economy and find that labor market frictions are the key factor for the gap. Furthermore, Shimer (2009) finds that search frictions combined with real wage stickiness endogenously produce the labor

[5] An indivisible labor decision plays a significant role in breaking the tight link between consumption and labor at both individual and aggregate levels, which also affects the volatility of the labor wedge (Chang and Kim, 2007).

wedge. All these papers are based on the representative-agent economy. I contribute to this literature by developing a model with heterogeneous agents to account for the two anomalies as in Chang and Kim (2007) and Takahashi (2014).

This study is also closely related and complementary to a chain of quantitative papers based on a heterogeneous-agent model for the two puzzles. Chang and Kim (2007) provide one of the first frameworks linking heterogeneity to the labor wedge. They develop a model economy of incomplete capital markets with discrete labor supply and idiosyncratic labor productivity shocks, and show that the measured labor wedge arises endogenously due to rich heterogeneity across agents and incomplete markets.[6] Takahashi (2015) builds a heterogeneous-agent DSGE model in the presence of time-varying wage risks. He finds that fluctuations in idiosyncratic wage risks resolve the hours-productivity puzzle and the large cyclical movement in the wedge. Differentiating from Chang and Kim (2007) and Takahashi (2014), the main contribution of this article is that I explicitly consider real wage rigidities, including long-term wage contracts, in a heterogeneous-agent DSGE model.

The remainder of this paper is organized as follows. Section 2 summarizes the two puzzles using U.S. aggregate data. In Section 3, I will build an incomplete asset market model with real wage rigidities and heterogeneous agents. The parameter values are determined in Section 4. Section 5 summarizes the findings of the model economies. In Section 6, I examine the role of heterogeneity in the two anomalies. Section 7 concludes.

2.2 Empirical Facts: Two Puzzles

In this section, I summarize empirical evidence regarding the two puzzles using U.S. aggregate data spanning from 1964:I to 2011:IV. Other than total hours of work, U.S. macroeconomic data used in this paper are standard as in the business cycle literature.[7] Since I believe that choosing a measure for hours worked is most important to compute reliable

[6]Takahashi (2014) finds that there are errors in the computational method of Chang and Kim (2007), and their model economy actually generates the strong correlation between productivity and hours and the less volatile labor wedge.

[7]See Subsection A1 in the appendix below for details of the data sources.

aggregate productivity and the labor wedge, I use two types of datasets for hours of work: the household survey (HS) and the establishment survey (ES).[8] The establishment survey is taken from the Current Employment Statistics (CES), which is conducted by the Bureau of Labor Statistics (BLS). The source for the household-level survey data is Cociuba, Prescott and Ueberfeldt (2009). They use the Consumer Population Survey (CPS) to compute total hours worked.

2.2.1 Puzzle 1: Low Correlation between Productivity and Hours

The relation between the aggregate labor productivity and aggregate hours is summarized in Figure 2.1. The aggregate labor productivity is computed as Y/H where Y is real gross domestic production (GDP) in the private business sector, and H is aggregate hours of work. The upper panel of Figure 2.1 exhibits the correlation between productivity and hours, which are computed with the household survey. As observed in the scatter plot in the top left panel, productivity does not seem to be correlated with hours. The correlation coefficient between the two macro variables is very small (0.23). The consistent pattern is also seen in the upper right panel, which shows that the cyclical components of productivity and hours over long periods of time. Hours often fluctuate in the opposite direction to productivity, which may cause the low correlation between the two time series. A similar relationship is also found in the bottom panel of Figure 2.1, where productivity and hours are computed using the establishment survey. The correlation coefficient between the two variables with the establishment survey is -0.06.

2.2.2 Puzzle 2: Large Volatile Labor Wedge

Following Chang and Kim (2007) and others, the labor wedge is defined as the gap between the marginal rate of substitution of aggregate leisure for aggregate consumption (MRS) and the marginal product of aggregate labor (MPL), i.e.,

[8]Similarly, Karabarbounis (2014) also uses both the household-level and the establishment-level survey data for hours of work when he computes the labor wedge.

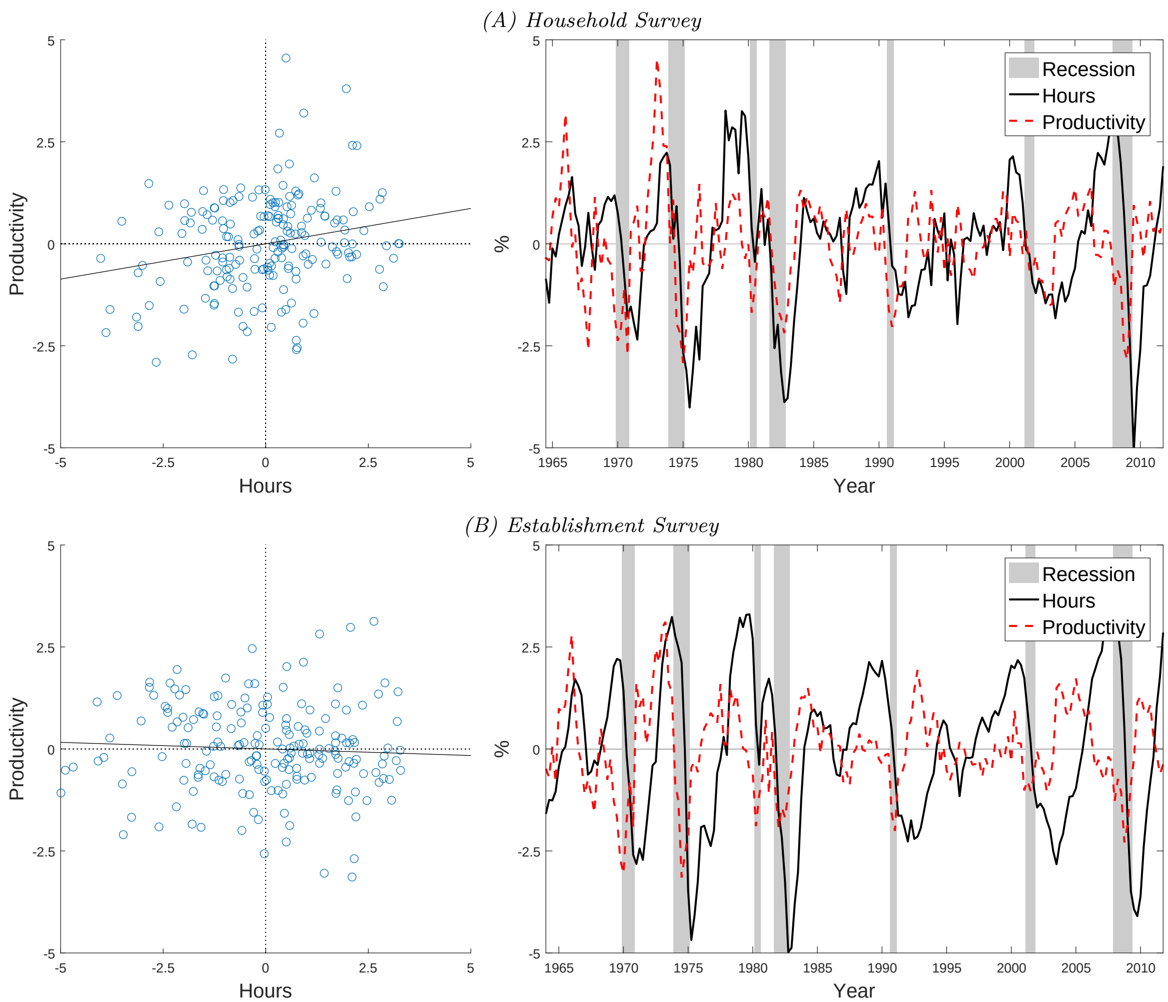

Figure 2.1: Hours and Productivity

Notes: All variables are logged and detrended by the Hodrick-Prescott filter with a smoothing parameter of 1,600.

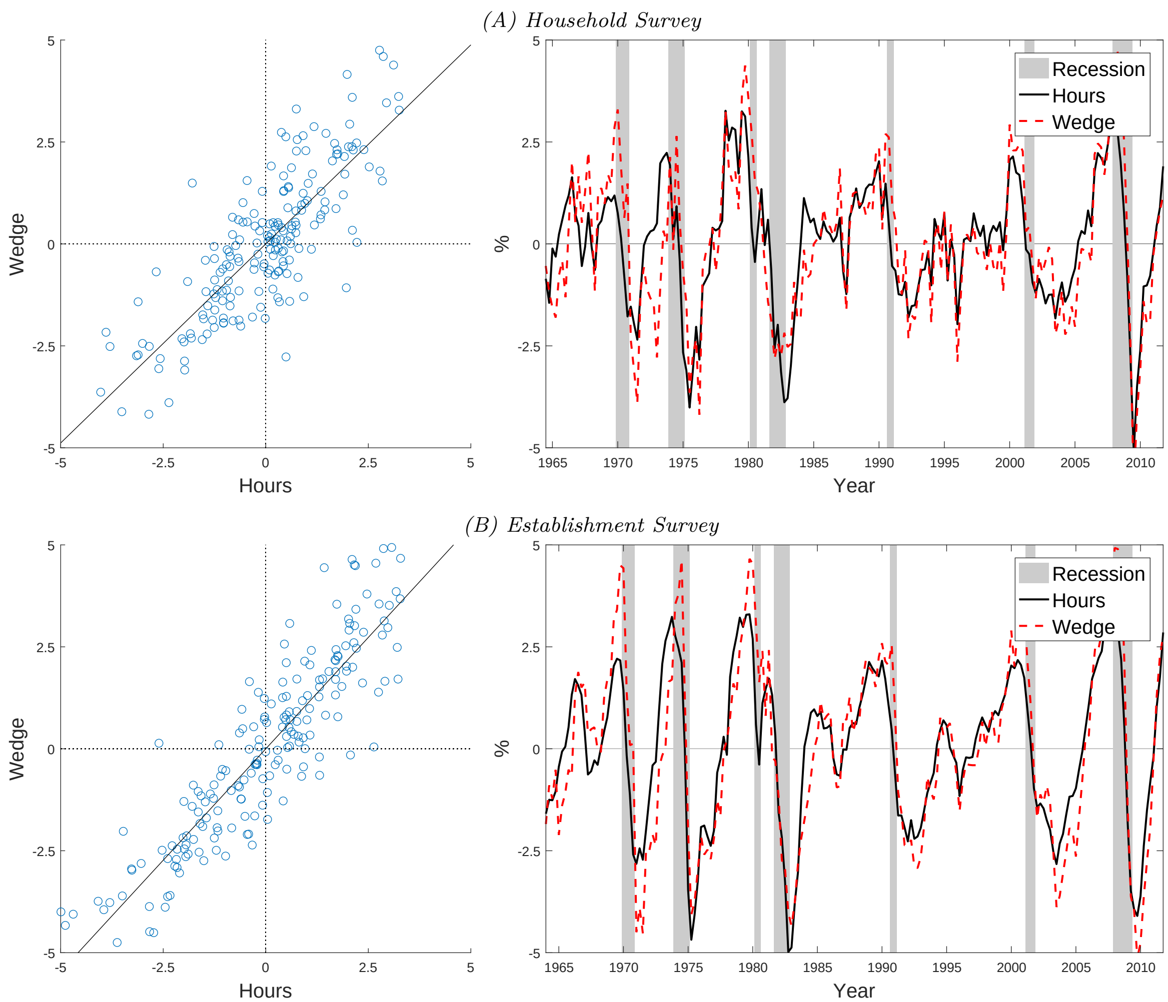

Figure 2.2: Hours and Labor Wedge

Notes: The labor wedge is computed from Equation (4.10). All variables are logged and detrended by the Hodrick-Prescott filter with a smoothing parameter of 1,600.

$$\log(wedge) = \log(CH^{\frac{1}{\chi}}) - \log(Y/H) \tag{2.1}$$

where C is aggregate expenditures on nondurable goods and services, and χ is aggregate labor supply elasticity.[9] $MRS(= CH^{\frac{1}{\chi}})$ is derived from the functional form of household's utility which will be discussed in Section 3. Figure 2.2 exhibits the relation between aggregate hours and the labor wedge. Regardless of what survey data I use, the correlation coefficient of the wedge with hours of work is high: 0.82 (0.90) in the household survey (in the establishment survey). More importantly, the labor wedge is highly volatile. The relative volatility of the wedge to Y is 0.86 (1.08) in the household survey (in the establishment survey), which is larger than that of aggregate hours of work, 0.72 (0.89). These findings are consistent with previous work, such as that of Chang and Kim (2007), Takahashi (2014) and Karabarbounis (2014).

To sum up, the correlation between aggregate productivity and hours worked is low and the labor wedge is large volatile. As discussed in Chang and Kim (2007), these two facts are closely related. According to Equation (4.10), the labor wedge arises since hours worked and productivity show a fairly low correlation. In the next section, I present a DSGE model economy with heterogeneous households and real wage rigidities to account for the two puzzles.

2.3 The Model

I build a simple dynamic stochastic general equilibrium (DSGE) model with a large population of heterogeneous households in labor productivity and with real wage rigidities. It is assumed that households cannot fully insure against individual productivity shocks, which follow a stochastic process. That is, capital markets are incomplete following Huggett

[9]It should be noted that micro labor supply elasticity is calibrated but the macro elasticity of labor supply is endogenously generated due to heterogeneity and indivisibility of labor supply. I choose $\chi = 1.5$, which is computed from steady state reservation wage distribution generated by the baseline model economy. The value is in the range of the estimates for the aggregate labor supply elasticity in the macro-labor literature.

(1993) and Aiyagari (1994). A labor supply decision for each household is assumed to be indivisible as in Hansen (1985) and Chang and Kim (2007). In particular, the model economy allows for the presence of real wage rigidities following Gali, Gertler and LopezSalido (2007) and Shimer (2009), who argue that labor market frictions, including real wage stickiness, are key factors for the labor wedge. As in Cho and Cooley (1995) and Cho, Cooley and Phaneuf (1997), real wage rigidities arise from wage contracts agreed to by households and firms. In these senses, the model economy in this paper is an extended version of Chang and Kim (2007), who feature extensive margins of labor supply in a heterogeneous-agent model. It is also in line with Cho and Cooley (1995), Cho, Cooley and Phaneuf (1997), Gali, Gertler and LopezSalido (2007), and Shimer (2009) in that wage rigidities are explicitly considered in the model economy.

2.3.1 Real Wage Contracts

In this subsection, I discuss the wage contract rule. Related papers are Cho and Cooley (1995), Cho, Cooley and Phaneuf (1997), Janko (2007), and Janko (2008). It is assumed that real wage rigidities arise from wage contracts agreed to by households and firms. Consider a k-period wage contact, and suppose that the firms and the households are in period of $t-k$. I use a natural formulation of the contract wages as a weighted average of the expected wage and the spot wage. As in Janko (2007), wage contracts are partially set in a synchronized manner: the expected wages for period $t-k+1$ to t are determined at the end of period $t-k$, while the spot wage is determined in each period. Additionally, similar to Cho and Cooley (1995), I assume that both expected and spot wages are based on the forecasting function for wage rates and the law of motion for aggregate capital from the economy with no wage rigidities. Specifically, following Chang and Kim (2007), the law of motion for the aggregate capital, K, and the forecasting function for the market wage rate are assumed to take log-linear functions of K and aggregate productivity, Z:[10]

[10]For an equilibrium with aggregate dynamics, I follow the procedure suggested Krusell and Smith (1998): a very high precision can be obtained by approximating the distribution across characteristics of households (μ) using the first moment of it. Hence, the mean asset, K, is used in forecasting the law of motion for μ.

$$\log K_{t+1} = a_0 + a_1 \log K_t + a_2 \log Z_t, \tag{2.2}$$

$$\log w_t = b_0 + b_1 \log K_t + b_2 \log Z_t. \tag{2.3}$$

The coefficients for the law of motion and the forecasting function can be obtained from the market clearing economy, where there are no wage rigidities. The aggregate productivity, Z, follows an AR(1) process in logs:

$$\log Z_{t+1} = \rho_Z \log Z_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, \sigma_Z^2). \tag{2.4}$$

Suppose that the firms and the households are in period of $t-k+j$, where j is defined as follows:

$$j = \begin{cases} q & for\ q > 0 \\ k & for\ q = 0 \end{cases}, \tag{2.5}$$

where q is the remainder after division of t by k.[11] w^e_{t-k+j}, which is the expected wage rate in period $t-k+j$ conditional on the information available in period $t-k$, is computed as expected value of Equation (3.2) such that:

$$\log w^e_{t-k+j} = E\left[b_0 + b_1 \log K_{t-k+j} + b_2 \log Z_{t-k+j} | \Omega_{t-k}\right], \tag{2.6}$$

where $\Omega_{t-k} \equiv (K_{t-k}, Z_{t-k})$. Hence, the expected wages for k successive periods are set simultaneously at the end of period $t-k$. Next, following Cho and Cooley (1995), I assume

[11]Hence, $1 \leq j \leq k$.

that w^s_{t-k+j}, which is the wage rate determined in the spot market in period $t-k+j$, is defined as:

$$\log w^s_{t-k+j} = b_0 + b_1 \log K_{t-k+j} + b_2 \log Z_{t-k+j}. \tag{2.7}$$

That is, the spot wages are also determined using Equation (3.4) but based on the current information.[12] Finally, w^c_{t-k+j}, which is the contract wage rate in period $t-k+j$, is determined as a weighted average of w^e_{t-k+j} and w^s_{t-k+j}:

$$w^c_{t-k+j} = \lambda w^e_{t-k+j} + (1-\lambda) w^s_{t-k+j}, \tag{2.8}$$

where $0 \leq \lambda \leq 1$ is the index of real wage rigidity.

According to Equation (3.1) and (3.2), w^e_{t-k+j} is expressed by the function of K_{t-k} and Z_{t-k}, and w^s_{t-k+j} is determined by the current state variables (K_{t-k+j} and Z_{t-k+j}). Thus, the contract wage rate, w^c_{t-k+j}, is a function of four aggregate variables of K_{t-k}, Z_{t-k}, K_{t-k+j}, and Z_{t-k+j}. Intuitively, the contract wage depends on two parameters: the degree of wage rigidity, λ, and the length of wage contracts, k.

2.3.2 Representative Firm

The production technology for the representative firm is represented by the constant returns-to-scale Cobb-Douglas function:

$$F(K, L, Z) \equiv Z K^{\theta} L^{1-\theta},$$

where K, L and θ denote aggregate capital, aggregate effective labor, capital income share, respectively. Following Cho and Cooley (1995) and Cho, Cooley and Phaneuf (1997),

[12] If there is no wage rigidity in the model economy, the spot wage is the same as the market equilibrium wage.

given the wage contract rule, it is assumed that the representative firm decides how much aggregate effective labor it demands. In other words, the households cede the right to make decisions about aggregate labor to the firm. Hence, given the contract wage rate, the aggregate effective labor L is determined by firm's profit maximization condition. Of course, given w^c and the real rental price for capital r, the demand for capital is also determined by the first-order conditions. That is,

$$w^c = (1-\theta)Z(K/L)^{\theta},$$

and

$$r + \delta = \theta Z(K/L)^{\theta-1},$$

where δ is the capital depreciation rate.

2.3.3 Heterogeneous Households

Each household maximizes the expected lifetime utility:

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\left(\log c_t - \Psi\frac{h_t^{1+1/\eta}}{1+1/\eta}\right)\right],$$

where $0 < \beta < 1$ is the time discount factor, c_t is consumption, h_t is hours of work, $\Psi > 0$ is a parameter for disutility from working, and η is micro Frisch elasticity of labor supply. A labor choice by each household is assumed to be indivisible following Hansen (1985), Rogerson (1988), and Chang and Kim (2007). In other words, a household makes a decision to work for a fixed amount of hours ($h = \overline{h}$) or none ($h = 0$). Accordingly, there are two occupational choices in the model economy: an employed worker and a non-employed worker.

It is assumed that households face idiosyncratic labor productivity shocks, x, which evolves with transition probabilities $P_x(x'|x) = Pr(x_{t+1} = x'|x_t = x)$. Asset markets are incomplete following Huggett (1993) and Aiyagari (1994): households cannot issue any assets contingent on their future idiosyncratic risks, x. Borrowing is allowed for a household, but it is exogenous amount, $\overline{a}$. I assume that the aggregate shocks and the individual shocks are independent of each other. In addition, the idiosyncratic risks are assumed to follow a log-AR(1) process:

$$\log x' = \rho_x \log x + \varepsilon_x, \quad \varepsilon_x \sim N(0, {\sigma_x}^2).$$

Suppose that a household is in period $t-k+j$, where j is defined in Equation (3.5). For each household, the individual state is the vector $\gamma \equiv (a, x)$, where a denotes assets that a household holds. As mentioned above, since w^c_{t-k+j} is a function of μ_{t-k}, Z_{t-k}, μ_{t-k+j}, and Z_{t-k+j}, the aggregate state is the vector $\Gamma \equiv (\mu, Z, \mu_{-k}, Z_{-k}, j)$, where μ is a joint distribution of assets and idiosyncratic labor productivity across households, and $Z_{-k}(\mu_{-k})$ is aggregate productivity shock (the joint distribution) in period of $t-k$.[13] Notice that the future aggregate state is the vector $\Gamma' \equiv (\mu', Z', \mu_{-k}, Z_{-k}, j+1)$ when $j < k$ since the household's expectation is still based on information in period of $t-k$, while it is the vector $\Gamma' \equiv (\mu', Z', \mu, Z, 1)$ when $j = k$ since, in period $t+1$, the base period for the expectation will change from period of $t-k$ to period of t.

2.3.3.1 Employment Decision

Since households agree to cede the right to decide an aggregate efficient labor to the representative firm, they should provide effective labor that the firm demands, denoted by L^c, hereafter. If the economy is based on the representative-agent assumption, all identical households should supply the same amount of labor, L^c. However, in the heterogeneous-agent model economy, since each household has a different reservation wage, it is needed to have an additional assumption how different households allocate their hours differently to fulfill the wage contract. Hence, I assume that households are employed in ascending order by their reservation wage rates until they provide L^c. Specifically, suppose that $\widetilde{w}$

[13] Following Krusell and Smith (1998), μ will be approximated by K.

is the wage rate at which households endogenously provide effective labor of L^c given the aggregate state variables. A household, whose reservation wage rate is less than or equal to $\widetilde{w}$, should work. In other words, households make labor supply decisions under $\widetilde{w}$. Thus, even if employment decisions are made endogenously under $\widetilde{w}$, some are voluntarily (non-)employed while others are involuntarily (non-)employed because of the wage contracts. Of course, the size of the involuntary labor choices depends on the degree of wage rigidity or the length of wage contracts.

Let's consider a recursive equilibrium to charaterize $\widetilde{w}$ and labor supply choices for households, $h(\gamma,\Gamma)$. The value function for an employed worker $\widetilde{V}^W(\gamma,\Gamma)$, is:

$$\widetilde{V}^W(\gamma,\Gamma) = \max_{c,a'} \left\{ \log c - \Psi \frac{\overline{h}^{1+1/\eta}}{1+1/\eta} + \beta \mathbb{E}\left[\widetilde{V}(\gamma',\Gamma') \right] \right\}$$

subject to

$$c = \widetilde{w} x \overline{h} + (1+r)a - a', c \geq 0,\ a' \geq \overline{a},$$

and

$$\mu' = \Theta(\Gamma),$$

where Θ denotes a transition operator for μ, and $\overline{a}$ is a borrowing constrain that limits the fixed amount of debt.

The value function for a non-employed worker, denoted by $\widetilde{V}^N(\gamma,\Gamma)$, is:

$$\widetilde{V}^N(\gamma,\Gamma) = \max_{c,a'} \left\{ \log c + \beta \mathbb{E}\left[\widetilde{V}(\gamma',\Gamma') \right] \right\}$$

subject to

$$c = (1+r)a - a', c \geq 0,\ a' \geq \overline{a} \text{ and } \mu' = \Theta(\Gamma).$$

The household's employment decision $h(\gamma,\Gamma) \in \left\{0, \overline{h}\right\}$ and value function $\widetilde{V}(\gamma,\Gamma)$ are defined as:

$$\widetilde{V}(\gamma,\Gamma) = \max_{h\in\left\{0,\overline{h}\right\}} \left\{\widetilde{V}^W(\gamma,\Gamma),\widetilde{V}^N(\gamma,\Gamma)\right\}.$$

Of course, for all Γ, households provide $L^c(\Gamma)$ under $\widetilde{w}$:

$$L^c(\Gamma) = \int xh(\gamma,\Gamma)d\mu.$$

2.3.3.2 Consumption-savings Decision

The next step of a decision for a household is a consumption-investment decision. Given employment status $h(\gamma,\Gamma)$, decisions for consumption and savings are made under the contract wage w^c such that:

$$V(\gamma,\Gamma) = \max_{c,a'} \left\{\log c - \Psi\tfrac{h(\gamma,\Gamma)^{1+1/\eta}}{1+1/\eta} + \beta\mathbb{E}\left[V(\gamma',\Gamma')\right]\right\}$$

subject to

$$c = w^c xh(\gamma,\Gamma) + (1+r)a, c \geq 0,\ a' \geq \overline{a} \text{ and } \mu' = \mathit{\Theta}(\Gamma).$$

To sum up, employment decisions are made under $\widetilde{w}$ to supply the labor the firm demands, while consumption-investment decisions are under w^c. Notice that the real return to capital, r, is the same for the two decisions.

2.3.4 Definition of Equilibrium

A recursive competitive equilibrium is a set of inputs $\{K(\Gamma), L^c(\Gamma)\}$, a set of factor pricing functions $\{w^c(\Gamma), r(\Gamma), \widetilde{w}(\Gamma)\}$, a set of value functions $\{\widetilde{V}^W(\gamma,\Gamma),\widetilde{V}^N(\gamma,\Gamma),\widetilde{V}(\gamma,\Gamma),V(\gamma,\Gamma)\}$, a set of optimal decision rules $\{h(\gamma,\Gamma), c(\gamma,\Gamma), a'(\gamma,\Gamma)\}$, and a forecasting function $\mathit{\Theta}(\Gamma)$ such that:

1. Real wage contracts

2. Employment decisions: given $\widetilde{w}(\Gamma)$ and $r(\Gamma)$, the optimal employment decision rule $h(\gamma,\Gamma)$ solves the value functions $\widetilde{V}^W(\gamma,\Gamma)$,$\widetilde{V}^N(\gamma,\Gamma)$ and $\widetilde{V}(\gamma,\Gamma)$.

3. Consumption-saving decisions: given $h(\gamma,\Gamma)$, $w^c(\Gamma)$ and $r(\Gamma)$, the optimal decision rules $c(\gamma,\Gamma)$ and $a'(\gamma,\Gamma)$ solve the value function $V(\gamma,\Gamma)$.

4. The firms optimize: for all Γ, $F_L(K(\Gamma),L^c(\Gamma),Z) = w^c(\Gamma)$ and $F_K(K(\Gamma),L^c(\Gamma,Z) = r(\Gamma)+\delta$.

5. Capital markets clear: for all Γ, $K(\Gamma) = \int a d\mu$.

6. Aggregate behaviors are consistent with individual ones: $\Theta(\Gamma)$ is consistent with the actual law of motion implied by the optimal policy function $a'(\gamma,\Gamma)$.

2.4 Calibration

Table 4.1 summarizes parameter values used in the model economies. The parameter η, which corresponds to the micro elasticity of labor supply, is set to 0.4 based on the findings that estimates of the micro elasticity of labor supply are around $0-0.5$. However, since a labor supply decision is discrete, choosing any values of η does not affect simulation results. An extensive margin for the labor supply, $\overline{h}$, is chosen to be 1/3. I choose the time discount factor, β, and the disutility parameter of working, Ψ, to target a one percent quarterly return to capital and a 60 percent employment rate for employed workers.[14] The borrowing constraint, $\overline{a}$, is set to -2.0, following Chang and Kim (2007).[15]

For the individual labor productivity shock process, I choose $\rho_x = 0.929$ and $\sigma_x = 0.227$ following Chang and Kim (2007) which is estimated with the AR(1) wage process using the PSID for 1979-1992.[16] For aggregate productivity shocks, I simply choose $\rho_z = 0.95$ and $\sigma_z = 0.007$ following Kydland and Prescott (1982). The capital income share, θ, is 0.36, and the quarterly depreciation rate, δ, is 2.5 percent. Unfortunately, there is little empirical evidence on the index of real wage rigidity, given the wage contract rule. Hence, I choose a set of values for λ, which are 0, 0.3, 0.5 and 0.7. As far as the length of wage contracts, k, is concerned, I consider one-, four- and eight-period contracts, i.e., k =1, 4, and 8.

[14]Recent U.S. data such as the Panel Study of Income Dynamics (PSID) and the Survey of Consumer Finances (SCF) consistently report that the employment rates are around 60 percent.

[15]$-\overline{a}(=2)$ is around doubled quarterly average income of the model.

[16]Chang and Kim (2007) estimate the AR(1) process of the logged wage rate using the maximum-likelihood estimation (MLE) method of Heckman (1979) to fix the selection bias problem, where the wages of households who did not work are not observable in the data.

Table 2.1: Parameters of the Model Economy

Parameter	Value	Description
β	0.983125	Time discount factor
$\overline{h}$	1/3	Extensive margin for hours worked
η	0.4	Labor supply elasticity
Ψ	166.2	Parameter for disutility from working
ρ_x	0.929	Persistence of productivity shocks
σ_x	0.227	Standard deviation of productivity shocks
$\overline{a}$	-2.0	Borrowing constraint
θ	0.36	Capital income share
δ	0.025	Capital depreciation rate
ρ_Z	0.95	Persistence of aggregate productivity shock
σ_Z	0.007	Standard deviation of innovation to aggregate productivity

2.5 Findings

2.5.1 One-period Wage Contract Case

As a benchmark case, I first summarize the key findings of the model economy with one-period wage contract ($k = 1$) to study the role of wage rigidity indexes, λ.

2.5.1.1 Wage Rigidity and Hours-Productivity Correlation

The cyclical properties of the aggregate variables for model economies are summarized in Table 2.2. Comovements between output and other key variables, such as consumption and investment, are well replicated in all the model economies as in the standard RBC models. The main focus in this paper is on variables related to labor markets. Table 2.2 and Figure 2.3 show that the correlation coefficient of hours and productivity decreases as the index of wage stickiness increases. The correlation coefficient of the two macro series in the model economies when $\lambda = 0.3$ and $\lambda = 0.7$ are 0.20 and -0.55, respectively, while it is 0.81 in the market clearing economy ($\lambda = 0$). Thus, the model economies with small amounts of wage stickiness reproduce reasonably well the fact that hours of work are not strongly correlated with productivity. The intuition is simple. During expansions, the aggregate labor demand curve shifts to the right, but the contract wage and hours are not determined

Table 2.2: Cyclicality of Aggregate Variables

Variable	Data (HS)	Data (ES)	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0.7$
$\rho(Y, C)$	0.81	0.81	0.91	0.87	0.83	0.79
$\rho(Y, I)$	0.95	0.95	0.99	0.99	0.99	0.99
$\rho(Y, H)$	0.84	0.86	0.96	0.89	0.87	0.87
$\rho(H, Y/H)$	0.23	-0.06	0.81	0.20	-0.28	-0.55
$\rho(H, wedge)$	0.82	0.90	0.94	0.94	0.97	0.98

Notes: The labor wedge is computed from Equation (4.10). All variables are logged and detrended by the Hodrick-Prescott filter with a smoothing parameter of 1,600. $\rho(A, B)$ denotes the correlation between variables A and B. HS and ES denote the household survey and the establishment survey, respectively.

at an intersection between the labor supply and demand curves due to the wage contract. Hence, wage rigidities prevent the wage and hours from moving together along the labor supply curve with aggregate shocks.

2.5.1.2 Wage Rigidity and Volatility of Labor Wedge

Table 2.3 reports the volatilities of the aggregate variables for the U.S. economy and the corresponding variables simulated by model economies. The model economy without wage rigidities (the market clearing economy) shows output volatility of 1.26, explaining around 60 percent of cyclical variation of output in the U.S. data. The model economies with wage rigidities exhibit larger output variation: the volatility of output is 1.40 when $\lambda = 0.3$ and 1.67 when $\lambda = 0.7$. Sample statistics of other variables are similar to those in the standard RBC models: consumption and investment is around 30-40 percent and three times as volatile as output, respectively.

A distinguishing feature of the model economy is the labor wedge. The wage stickiness plays an important role in generating the large volatile wedge. According to Table 2.3 and Figure 2.3, the volatility of the wedge increases with the degree of wage stickiness: the volatilities of the wedge relative to output in the model economies when $\lambda = 0.3$ and $\lambda = 0.7$ are 0.66 and 1.27, respectively. However, the market clearing model economy fails to replicate the cyclical variation of the labor wedge in the U.S. economy: relative volatility of the wedge is 0.23 when $\lambda = 0$. According to Equation (4.10), the volatility of wedge is positively related

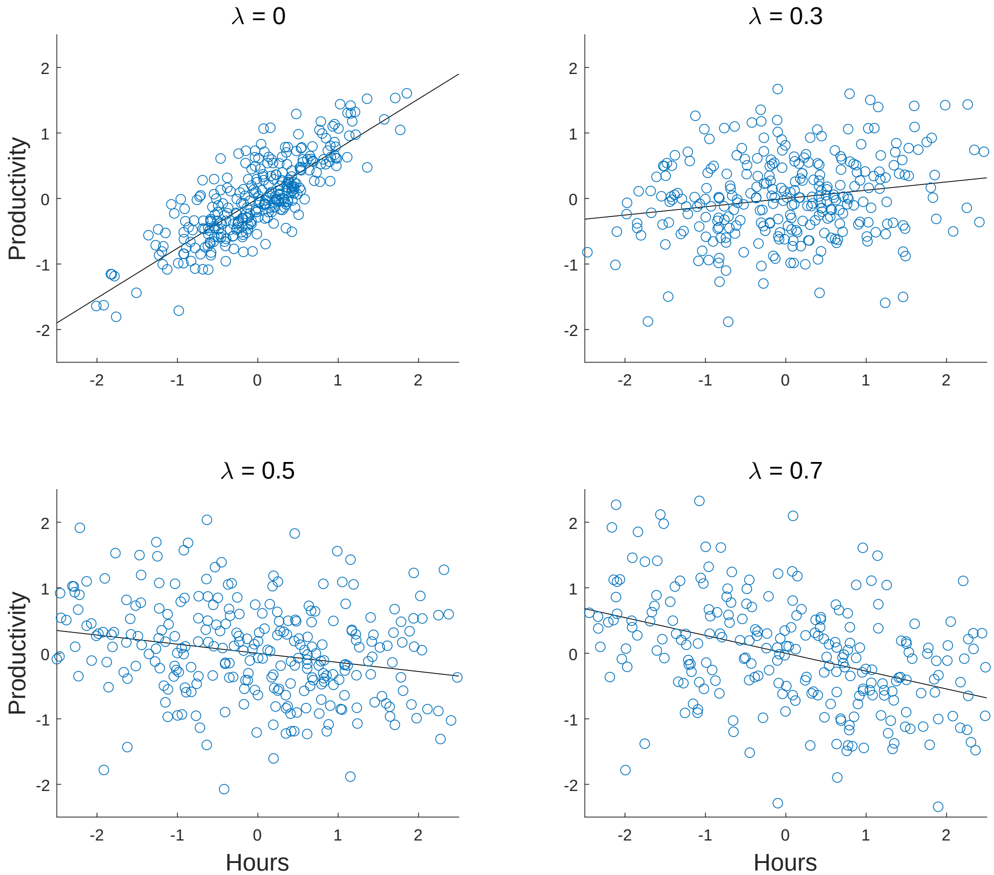

Figure 2.3: Correlation between Productivity and Hours

Variables are logged and detrended by the Hodrick-Prescott filter with a smoothing parameter of 1,600 for each λ.

Table 2.3: Volatilities of Aggregate Variables

Variable	Data (HS)	Data (ES)	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0.7$
σ_Y	2.11	2.11	1.26	1.40	1.53	1.67
σ_C/σ_Y	0.43	0.43	0.35	0.33	0.31	0.29
σ_I/σ_Y	2.67	2.67	3.00	3.11	3.20	3.28
σ_H/σ_Y	0.72	0.89	0.54	0.79	1.01	1.20
$\sigma_{Y/H}/\sigma_Y$	0.55	0.51	0.51	0.47	0.52	0.59
σ_{wedge}/σ_Y	0.86	1.08	0.23	0.66	1.00	1.27

Notes: The labor wedge is computed from Equation (4.10). All variables are logged and detrended by the Hodrick-Prescott filter with a smoothing parameter of 1,600. σ_A denotes the standard deviation (multiplied by 100) of a variable A. HS and ES denote the household survey and the establishment survey, respectively.

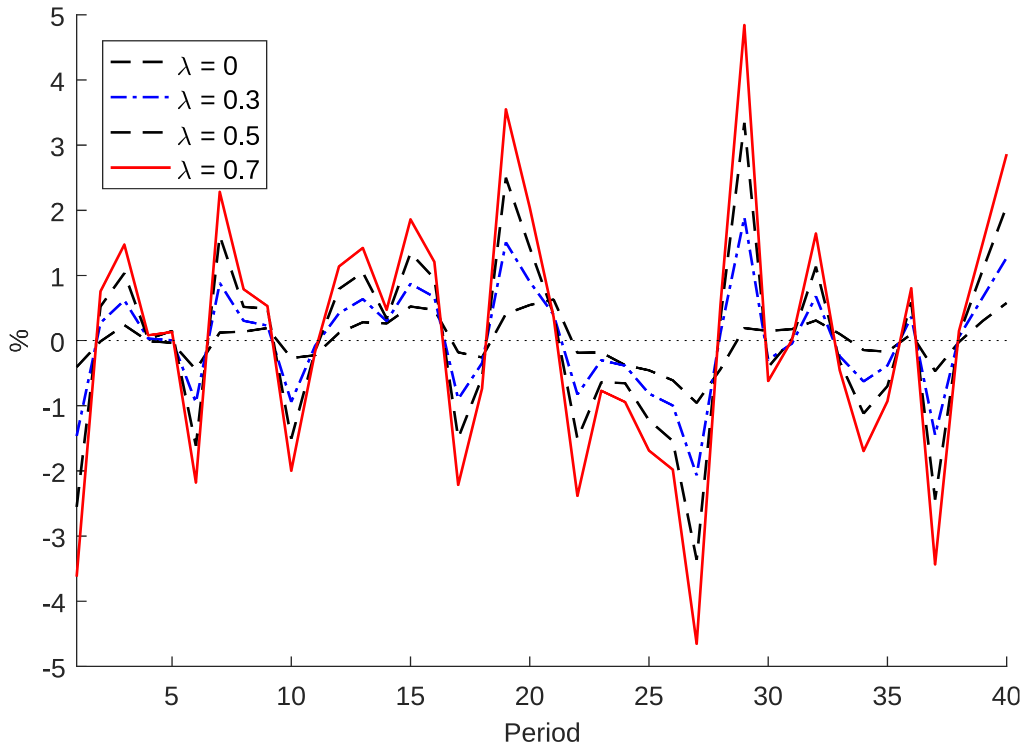

Figure 2.4: Volatility of Labor Wedge

The labor wedge is computed from Equation (4.10). The series is logged and detrended by the Hodrick-Prescott filter with a smoothing parameter of 1,600 for each λ.

to the cyclical movement in hours worked (H) and the volatility of productivity (Y/H), and negatively related to the correlation between hours and productivity. Firstly, the relative volatility of aggregate hours worked gets larger with the degree of wage rigidity: the cyclical variation of hours of work relative to output is 0.54 and 1.20 with wage rigidity indexes of 0.3 and 0.7, respectively. Second, interestingly, there are small differences in the cyclical variation of productivity relative to output across model economies with different λ. Lastly, as found earlier in Table 2.2, the correlation between hours and productivity decreases as the index of wage rigidity increases. From the three findings above, I conclude that large volatile hours worked and the low correlation coefficient between hours and productivity produce the large cyclical movement in the wedge.[17]

[17]The correlation coefficient of the wedge with aggregate hours worked is also reasonably replicated: it is large in the model economies as in the U.S. data (see Table 2.2).

Table 2.4: Volatilities and Cyclicality of Aggregate Variables: Long-period Wage Contracts

Variable	Data (HS)	Data (ES)	$k=0$	$k=1$	$k=4$	$k=8$
σ_Y	2.11	2.11	1.26	1.40	1.56	1.69
σ_C/σ_Y	0.43	0.43	0.35	0.33	0.31	0.30
σ_I/σ_Y	2.67	2.67	3.00	3.11	3.20	3.27
σ_H/σ_Y	0.72	0.89	0.54	0.79	0.97	1.06
$\sigma_{Y/H}/\sigma_Y$	0.55	0.51	0.51	0.47	0.43	0.41
σ_{wedge}/σ_Y	0.86	1.08	0.23	0.66	0.90	1.01
$\rho(Y,C)$	0.81	0.81	0.91	0.87	0.83	0.80
$\rho(Y,I)$	0.95	0.95	0.99	0.99	0.99	0.98
$\rho(Y,H)$	0.84	0.86	0.96	0.89	0.91	0.92
$\rho(H,Y/H)$	0.23	-0.06	0.81	0.20	-0.15	-0.33
$\rho(H,wedge)$	0.82	0.90	0.94	0.94	0.98	0.98

Notes: All variables are logged and detrended by the Hodrick-Prescott filter with a smoothing parameter of 1,600. σ_A denotes the standard deviation (multiplied by 100) of a variable A and $\rho(A,B)$ denotes the correlation between variables A and B. HS and ES denote the household survey and the establishment survey, respectively. $\lambda = 0.3$ for all k other than $k = 0$. $k = 0$ denotes the absence of wage rigidities in the model economy.

2.5.2 Multi-period Wage Contract Case

In the benchmark case above, the length of wage contracts is a quarter. However, in reality, the contract periods are much longer: four or eight quarters. Hence, I examine how the cyclical behavior of the economy varies with the length of the wage contracts, focusing on the two puzzles. In fact, the long-term wage contracts play a significant role in accounting for the business cycle properties of the economy. For example, Cho and Cooley (1995) find that the length of wage contracts is important in the cyclical variation of aggregate variables. Cho, Cooley and Phaneuf (1997) also quantitatively estimate the welfare cost of nominal wage contracting across the contract periods and find that the welfare costs can vary over the contract length.

Summary statistics of the aggregate variables simulated by model economies over contract periods are summarized in Table 2.4. Given the wage rigidity index ($\lambda = 0.3$), the longer wage contract periods generate the larger volatile labor wedge and the lower correlation coefficients between productivity and hours. The relative volatility of the labor wedge is 0.90 in the four-period wage contract and around one in the eight-period wage contract,

Table 2.5: Required Index of Wage Rigidity to Replicate the U.S. Data

	Four-period Contract	Eight-period Contract
$\rho(H, Y/H)$	0.26	0.22
σ_{wedge}/σ_Y	0.35	0.29

Notes: The labor wedge is computed from Equation (4.10). All variables are logged and detrended by the Hodrick-Prescott filter with a smoothing parameter of 1,600.

while it is 0.66 in the short-term wage contract rule ($k = 1$). The correlation coefficients between productivity and hours in the long-term wage contracts are small negative numbers, while the correlation is 0.20 in the short-term wage contract. These results imply that the length of wage contracts plays a role in implicitly generating stickier wages.

Next, an interesting question is how large index of wage stickiness is required to solve the two puzzles when long-term wage contracts are considered. Table 2.5 reports the required degree of the wage stickiness to reproduce the correlation between hours and productivity and the relative volatility of the labor wedge of the U.S. economy.[18] According to the upper panel of the Table 2.5, the required index of wage rigidity is around $0.20 - 0.25$ to obtain the zero correlation coefficient. As far as the volatility of the labor wedge is concerned, the second row of the Table 2.5 suggests that around $0.30 - 0.35$ of the wage rigidity index is needed to replicate the average of the volatility of the wedge relative to output in the U.S economy, which is around one. Therefore, I argue that a small amount of real wage rigidity would be enough to reproduce the low correlation between productivity and hours and the large volatility of the labor wedge when the long-term wage contracts are considered in the heterogeneous-agent model. These findings are consistent with empirical work on real wage rigidity supporting the evidence that the degree of real wage rigidity in the U.S. is small (Dickens et al., 2007; Holden and Wulfsberg, 2009; Deelen and Verbeek, 2015).[19]

[18] I use the cubic spline interpolation method to approximate the relative volatility of the labor wedge and the correlation between hours and productivity with five grid points for λ. Then I compute the required λ which replicates the U.S. data.

[19] These results are also in line with work of Cho and Cooley (1995), who show that output volatility measured in the U.S. data is replicated with a very small amount of wage rigidity when a long-term wage contract is considered.

2.6 Role of Heterogeneity

In some previous related work based on the representative-agent (RA) economies, the labor wedge arises when other features are introduced into the model economy (Benhabib, Rogerson and Wright, 1991; Hall, 1997; Gali, Gertler and LopezSalido, 2007; Shimer, 2009). Not only that, previous work in the context of the representative-agent models with wage rigidity—Cho and Cooley (1995) for example—has also successfully reproduced the fact that hours of work are not highly correlated with labor productivity.[20] Therefore, it seems natural to ask why heterogeneity is necessary and what the role of heterogeneity is in the two puzzles. To investigate the importance of heterogeneity for resolving the two anomalies, I compare the key business cycle properties of the representative- and heterogeneous-agent (HA) model economy, controlling for wage rigidities. The basic assumptions for the wage contacts in the RA model are the same as those in the HA model other than the employment decisions for households. Since households are identical in the RA model, they cannot decide who works or not as in the HA model. Hence, given the contract wage, the representative household is assumed to provide L^c, which is determined by firm's profit maximization condition.[21] For calibration in the RA model, I choose $\beta = 0.99$ to target a one percent quarterly return to capital, and I set the disutility parameter of working, Ψ, to match the aggregate hours of 0.2. Importantly, the labor supply elasticity, η, is chosen to be 1.5, which is the same as the aggregate labor supply elasticity generated by the HA model.[22] Other remaining parameters are the same as those in the HA model.

Summary statistics of the aggregate variables simulated by the RA model economies over the indexes of wage rigidity and the length of wage contracts are summarized in Table 2.6. As in the HA model economy, in the RA model, the correlation coefficient of hours

[20]Cho and Cooley (1995) yield a very small correlation between productivity and hours using a model with nominal wage contracts and monetary shocks.

[21]The RA model in this paper is similar to the model of Cho and Cooley (1995), but Cho and Cooley (1995) consider nominal wage rigidities with monetary policies while this work introduces real wage rigidities without nominal shocks.

[22]In a heterogeneous-agent model economy with indivisible labor, macro labor supply elasticity can be computed from steady state reservation wage distribution. See Chang and Kim (2006) for details.

Table 2.6: Volatilities and Cyclicality of Aggregate Variables: RA Model

Variable	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.5$	$\lambda = 0.7$	$k = 4$	$k = 8$
σ_Y	1.27	1.39	1.51	1.64	1.54	1.65
σ_C/σ_Y	0.35	0.55	0.64	0.73	0.51	0.49
σ_I/σ_Y	3.00	2.41	2.21	2.02	2.65	2.75
σ_H/σ_Y	0.42	0.60	0.73	0.84	0.70	0.75
$\sigma_{Y/H}/\sigma_Y$	0.60	0.50	0.46	0.43	0.43	0.38
σ_{wedge}/σ_Y	0.00	0.66	0.98	1.25	0.70	0.74
$\rho(Y, C)$	0.92	0.96	0.95	0.94	0.90	0.87
$\rho(Y, I)$	0.99	0.98	0.96	0.94	0.97	0.97
$\rho(Y, H)$	0.95	0.92	0.91	0.91	0.93	0.95
$\rho(H, Y/H)$	0.95	0.65	0.40	0.17	0.55	0.52
$\rho(H, wedge)$	0.00	0.90	0.94	0.96	0.92	0.93

Notes: All variables are logged and detrended by the Hodrick-Prescott filter with a smoothing parameter of 1,600. σ_A denotes the standard deviation (multiplied by 100) of a variable A and $\rho(A, B)$ denotes the correlation between variables A and B. $k = 1$ for all λ other than $\lambda = 0$, and $\lambda = 0.3$ for all k.

and productivity decreases, and the volatility of the wedge increases as the degree of wage stickiness or the length of wage contracts increases. However, the effect of wage stickiness is much less in the RA model. For example, when $\lambda = 0.3$ with $k = 4$, in the RA model, the volatility of the wedge is 0.70, and the correlation of hours and productivity is 0.55, while they are 0.95 and -0.15, respectively, in the HA model economy. Not only that, according to Table 2.7, the required degree of the wage stickiness under the RA model with long-term wage contracts, which is able to reproduce the two key business cycle moments of the U.S. economy, is around $0.5 - 0.7$, whereas it is around $0.20 - 0.35$ in the HA model as in Table 2.5. From these counterfactual analyses, I conclude that heterogeneity plays an important role in solving the two puzzles. First, heterogeneity allows the labor supply curve to evolve over time since reservation wage distribution or wealth distribution is time-varying in response to the aggregate shocks in the presence of heterogeneity (Chang and Kim, 2007). As a result, heterogeneity also plays a role in breaking the positive linear relation between productivity and hours and, in turn, generate the low correlation between the two time series. Furthermore, as found in Chang and Kim (2007), the labor wedge arises endogenously in the presence of heterogeneity. Table 2.3 and Figure 2.3 present that the volatility of the wedge is

Table 2.7: Required Index of Wage Rigidity to Replicate the U.S. Data: RA Model

	Four-period Contract	Eight-period Contract
$\rho(H, Y/H)$	0.65	0.57
σ_{wedge}/σ_Y	0.50	0.46

Notes: The labor wedge is computed from Equation (4.10). All variables are logged and detrended by the Hodrick-Prescott filter with a smoothing parameter of 1,600.

positive in the HA model even if there is no wage rigidity in the labor market ($\lambda = 0$), while it is zero in the RA model when $\lambda = 0$.[23] As mentioned above, the empirical evidence, found by Dickens et al. (2007), Holden and Wulfsberg (2009), and Deelen and Verbeek (2015), suggests that the size of real wage rigidity in the U.S. economy is small. Therefore, I argue that the HA model with wage rigidities is compatible with the empirical findings while the RA model may not.

2.7 Conclusion

This paper studies the quantitative implications of real wage rigidities and heterogeneity for the two long-standing puzzles in the business cycles literature, the weak comovement of hours worked with labor productivity and the large cyclical movement in the labor wedge. I shed light on these issues by extending a heterogeneous-agent model with an indivisible labor supply choice to real wage rigidities.

The main findings of this paper can be summarized as follows. I find that the correlation coefficient between hours and productivity decreases, and the volatility of the labor wedge increases when the index of wage stickiness or the length of wage contracts increases. Heterogeneity also plays a role in solving the two puzzles since heterogeneity allows the aggregate labor supply curve to move in response to aggregate productivity shocks and the wedge to be endogenously produced. From these results, I argue that a small amount of real wage stickiness would be sufficient to resolve both anomalies when long-term wage contracts and heterogeneity are taken into account.

[23] Of course, the indivisibility of a labor decision also helps to solve the puzzles. See Chang and Kim (2007) for the role of the indivisible labor.

3. THE HETEROGENEOUS RESPONSES OF CONSUMPTION BETWEEN POOR AND RICH TO GOVERNMENT SPENDING SHOCKS*

3.1 Introduction

Understanding how government spending shocks affect an economy is important since fiscal instruments are often used to smooth economic fluctuations, and they may directly affect consumers' welfare. Existing papers in the literature have devoted a great deal of effort to finding the effects of government expenditure on real economic activity.[1] However, most of them have mainly focused on how government spending affects macro variables, such as aggregate output, consumption, and employment. In addition to the aggregate effects of a public spending shock, the distributional effects are also a central issue in public debates, when considering increasing concerns about economic inequality. Accounting for this issue is also of importance to the discussion of economic policies since it is closely related to stabilization and redistributive policies.

In fact, government spending shocks have substantially different impacts on different consumers. Figure 3.1 exhibits the responses of average non-durable consumption across income quintiles to government spending shocks using the Consumer and Expenditure Survey (CEX), which spans from the first quarter of 1980 to the third quarter of 2008.[2] I use the Survey of Professional Forecasters (SPF) forecast errors to identify government spending policy shocks following Ramey (2011) and Anderson, Inoue and Rossi (2016).[3] In order to study the effects of a government spending shock, I consider a three-variable Vector

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[1] For example, empirical studies such as Blanchard and Perotti (2002), Gali, Lopez-Salido and Valles (2007) and Zubairy (2010) find significantly positive responses of output and consumption to an increase in government spending.

[2] I exclude zero lower bound (ZLB) periods and onward, but I find that estimation results for the full sample (1980:I-2015:III) are still robust. See appendix for details.

[3] The measure of SPF forecast errors is defined as the difference between actual federal spending growth and the one-quarter-ahead SPF forecasted growth.

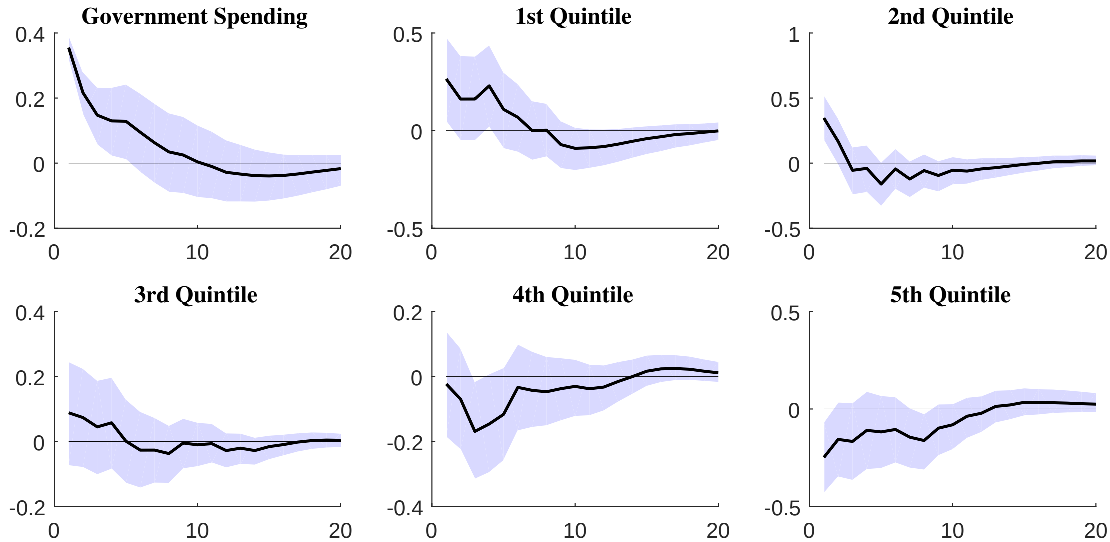


Figure 3.1: Responses of Consumption across Income Quintiles

Note: Responses of logged real per capita government spending and logged average real per capita consumption across the income quintiles to a government spending shock. The sample periods are 1980:I-2008:III. "1st Quintile" denotes the lowest income quintile, and "5th Quintile" denotes the top income quintile. The shaded regions are the 68 percent confidence bands generated by Monte Carlo simulations.

Autoregressive (VAR) model including the SPF forecast errors, government spending, and consumption.[4] As shown in Figure 3.1, there are substantial differences in the responses of consumption across income groups to spending shocks: consumption increases for the poor while it decreases for the rich when government expenditure rises. The peak multipliers for the first three quintiles are positive while they are negative for the top two quintiles.[5] For example, the peak multiplier for the lowest income quintile is 0.15 whereas that of the highest is -0.41. In other words, when government increases its spending by one dollar, the poorest consumers increase their consumption by 15 cents, but consumption for the richest decreases by 41 cents on average.[6] These empirical results are consistent with those in Anderson, Inoue and Rossi and De Giorgi and Gambetti (2012).

However, the theoretical background for these empirical findings is unclear. As discussed

[4]See appendix for details on the data and the estimation approach.

[5]The peak multiplier for a group j is computed as $max_h sign\left(\frac{\Delta log C^j_{t+h}}{\Delta log G_t}\right)\left|\frac{\Delta log C^j_{t+h}}{\Delta log G_t}\right|\frac{\overline{C^j}}{\overline{G}}$ where $\frac{\overline{C^j}}{\overline{G}}$ is the average ratio of consumption of the group j to government spending.

[6]These findings are robust when the cumulative multipliers are used as a measure of the effects of government spending shocks. The cumulative multiplier is defined as the sum of the responses of consumption divided by the sum of changes in government expenditure.

by Anderson, Inoue and Rossi (2016), the behavior of the poor and the rich can be respectively explained by different theories. For example, the responses of individuals in the lower income groups can be supported by traditional Keynesian theory such as IS-LM models, while the behavior of the rich can be explained by Neoclassical growth theory including Real Business Cycle (RBC) models.[7] Therefore, this study tries to explain why government expenditure shocks affect consumers differently using a quantitative analysis based on a heterogeneous agent economy.

To this aim, I build a simple dynamic stochastic general equilibrium (DSGE) model where there are a large population of heterogeneous households, a government, and many identical firms. The model economy can be characterized by four main features. First, a household cannot fully insure against idiosyncratic productivity shocks that it faces: the asset market is incomplete as in Huggett (1993) and Aiyagari (1994). Second, it is assumed that a labor supply decision for a household is indivisible following Hansen (1985) and Chang and Kim (2007). Third, and more importantly, the model economy allows for a progressive taxation scheme following Persson (1983), Benabou (2002), Heathcote, Storesletten and Violante (2017) and Ferriere and Navarro (2017). Lastly, government spending is productive as in Barro (1990) and Baxter and King (1993): government spending is in the production function, so firms' productivity is positively affected by government activity. With this assumption, expansionary government spending shocks can increase labor demand, which is one of the important channels to generate reasonable responses of key aggregate and disaggregate variables.

The model economy in which government spending is financed by a change in tax progressivity successfully replicates the different responses of consumption between the poor and the rich to government spending shocks. In the model economy, when the government increases its spending unexpectedly, there are two main effects on after-tax wage rates, which are key

[7]In the Neoclassical growth theory such as the RBC model, consumers lower their consumption after a positive government spending shock mainly due to a negative wealth effect induced by an increase in current or future taxes. On the other hand, according to the textbook IS-LM theory, households increase their consumption in response to a positive spending shock since they behave in a non-Ricardian fashion.

factors in determining households' consumption and employment.[8] One is an increase in tax progressivity to finance government expenditure, and the other is a rise in labor demand induced by productive government spending. The former has different impacts on after-tax wages across individual income levels while the latter is an economy-wide positive effect on post-tax wages. Specifically, after-tax wage rates rise for consumers in the bottom income quintiles since an increase in wage rates induced by productive government spending dominates a slight rise in tax rates, while post-tax wage rates fall for the top income groups since the positive wage effect cannot fully offset a significant increase in income tax rates. Therefore, consumption for the poor increases since after-tax wage rates increase while the rich decrease their consumption because of a fall in post-tax income. Indivisibility of the labor supply decision helps the poor increase their consumption since they are employed in response to positive government spending shocks, and their labor income increases substantially. I also provide supportive empirical evidence for the key channels of the model. First, I empirically document that tax progressivity rises in response to positive public spending shocks. Second, I also find empirical evidence that poor households increase employment while employment rates for the rich tend to decrease in response to an unexpected rise in government spending.

The remainder of this paper is organized as follows. The previous studies related to this work are summarized in Section II. In Section III, I build an incomplete asset market model that employs a progressive tax system, indivisible labor, and productive government spending. Sections IV and V summarize the key findings of the model economy. Section VI concludes.

3.2 Related Literature

The main contribution of this paper is twofold. First, this study contributes to the literature by providing a theoretical background for the observed distributional effects of

[8]As discussed in Hall (2009), the intratemporal optimality condition is a key to account for the effect of the government expenditure on consumption.

government spending shocks. De Giorgi and Gambetti (2012) and Anderson, Inoue and Rossi (2016) study empirical evidence for the heterogeneous effects of an unexpected government spending shock on consumers. Using the CEX data, De Giorgi and Gambetti employ the common components of the consumption deciles in a VAR with a government spending shock. Their main result is that consumption increases at the bottom of consumption distribution while it falls at the top after a government spending shock. Using a VAR with the SPF forecast errors, Anderson, Inoue and Rossi also find that unexpected increases in government spending hurt the working-age and the wealthiest individual the most in terms of consumption. These empirical findings can be replicated by the model economy. Other related empirical work is Owyang and Zubairy (2013) and Giavazzi and McMahon (2012).[9]

Second, this study is complementary to a chain of quantitative papers incorporating heterogeneity across individual households to account for the effects of fiscal policies. Key contributions are Heathcote (2005), Ferriere and Navarro (2017), and Gali, Lopez-Salido and Valles (2007). Heathcote (2005) provides one of the first frameworks linking a fiscal policy with heterogeneous agents. He develops a model economy of incomplete capital markets to analyze how changes in the timing of linear taxes for income affect an economy in the short run. The main result of his work is that when capital markets are incomplete, income tax cuts affect consumption largely compared to the complete market economy. Heathcote focuses on the impacts of tax shocks on individual consumption whereas this paper considers the effects of government spending shocks on consumers. The model economy in this study is probably closest to that of Ferriere and Navarro. They adopt an incomplete market model which incorporates indivisible labor and progressive taxes to assess the effects of government spending. From the simulation result of the model, they argue that when government expenditure is financed with a more progressive taxation scheme, a multiplier on aggregate

[9]Owyang and Zubairy (2013) investigate the effects of federal government spending shocks on state-level variables and find significant variation in the responses of state-level personal income and employment. Using household-level data, including the Panel Study of Income Dynamics (PSID) and the CEX, Giavazzi and McMahon (2012) also study how individuals respond to a change in government expenditure and find significant heterogeneous responses of hours, consumption, and real wages across households to a spending shock.

consumption can be positive. While Ferriere and Navarro mainly focus on the responses of aggregate variables by asking how government spending can be expansionary, this paper focuses more on the disaggregate effects of government expenditure on consumption across income groups. Relative to the government spending analysis of Ferriere and Navarro, I introduce an explicit role of government spending in the form of productive government spending, which is also an important feature to account for the reasonable responses of key macro and micro variables.[10] Gali, Lopez-Salido and Valles (2007) find that sticky prices and constrained consumers (also called rule-of-thumb or non-Ricardian consumers) in the context of calibrated New Keynesian models can reproduce an increase in aggregate consumption in response to a rise in public expenditure.[11] While there are only two types of heterogeneity in the model economy of Gali, Lopez-Salido and Valles, the model economy in this paper generates rich heterogeneity across households' wealth, income, and consumption. Compared to the work of Gali, Lopez-Salido and Valles, another contribution of this study is that the model economy can successfully account for the responses of both aggregate and disaggregate consumption to government spending shocks without introducing non-Ricardian consumers. Other studies analyzing relevance of heterogeneity for fiscal policies are Kaplan and Violante (2014), Heathcote, Storesletten and Violante (2017), and Chang, Chang and Kim (2018).

3.3 The Model

I develop a simple dynamic stochastic general equilibrium (DSGE) model which employs a continuum (measure one) of heterogeneous households, a government, and many identical firms. In the model economy, there are four main assumptions. First, asset markets are incomplete as in Huggett (1993) and Aiyagari (1994) in that households cannot fully insure

[10]For example, with this assumption, the model economy can generate an increase in the wage rate in response to positive government spending shocks, while the standard RBC model predicts a decrease in wages.

[11]In the model of Gali, Lopez-Salido and Valles (2007), constrained households are assumed to behave in a "hand-to-mouth" manner. In other words, they completely consume their labor income earned in the current period, and they cannot intertemporally optimize their consumption. Hence, constrained consumers tend not to smooth their consumption path in the face of fluctuations in taxes.

against their idiosyncratic productivity shocks. This assumption helps generate substantial heterogeneity across characteristics of individual households including wealth, income, employment status, and consumption. Second, following Hansen (1985), Rogerson (1988) and Chang and Kim (2007), it is assumed that a household indivisibly decides hours of work.[12] Third, more importantly, the model economy allows for a progressive taxation scheme for the government following Persson (1983), Benabou (2002), Heathcote, Storesletten and Violante (2017) and Ferriere and Navarro (2017). Lastly, government expenditure is productive in the production of firms: the firms' production is affected by productive government activity (Barro 1990, Baxter and King 1993, Fisher and Turnovsky 1995, Leeper, Walker and Yang 2010). The different tax burden helps consumers have different after-tax wage rates across their income, which is a main source of the heterogeneous responses of consumption across the income distribution. In particular, an interaction between productive public spending and a discrete labor decision allows poor households to be employed, and hence their consumption increases in response to a rise in government expenditure. Thus, I contribute to the literature by incorporating productive public expenditure and progressivie taxation into a heterogeneous agent model economy.

3.3.1 Environment

3.3.1.1 Households

Each household maximizes her expected lifetime utility over consumption c_t and hours of work h_t, shown as :

$$\max_{\{c_t,h_t\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\ln c_t - \chi \frac{h_t^{1+1/\phi}}{1+1/\phi} \right) \right]$$

subject to

[12] As is well-known, extensive margins for time devoted to work play an important role in accounting for the variation in total hours worked.

$$c_t + a_{t+1} = w_t x_t h_t + (1 + r_t)a_t - T(w_t x_t h_t + r_t a_t),$$

$$c_t > 0 \text{ and } a_{t+1} \geq \bar{b},$$

where $0 < \beta < 1$ denotes the time discount factor, $\chi > 0$ is a parameter for disutility from working, and ϕ represents labor supply elasticity. When a household works for h_t amount of hours, she earns $w_t x_t h_t$ as wage earnings, where w_t is the wage rate for the efficiency unit of labor, and x_t denotes her labor productivity. A household can save or borrow by trading a claim for assets a_t, which yields the real rate of return, r_t. A household faces a borrowing constraint that limits the fixed amount of debt: the assets holding cannot go below $\bar{b}$ at any time. Each household should pay taxes T, which depend on her total income (the sum of labor and capital income), $w_t x_t h_t + r_t a_t$.[13] Importantly, it is assumed that a labor supply decision made by a household is indivisible following Hansen (1985), Rogerson (1988), and Chang and Kim (2007): a household supplies a fixed amount of hours ($h_t = \bar{h}$), or she does not work at all ($h_t = 0$). Accordingly, there are two employment statuses for each household: employment and non-employment. The capital markets are incomplete following Huggett (1993) and Aiyagari (1994): households cannot issue any assets contingent on their future idiosyncratic risks x, which follows a stochastic process with transition probabilities $P_x(x'|x) = Pr(x_{t+1} = x'|x_t = x)$. In addition, it is assumed that the idiosyncratic risks to productivity follow an AR(1) process in logs:

$$\ln x' = \rho_x \ln x + \varepsilon_x, \quad \varepsilon_x \sim N(0, {\sigma_x}^2).$$

3.3.1.2 Firms

The production technology for the representative firms is represented by the function given by:

$$F(K, L, G) \equiv K^\alpha L^{1-\alpha} G^\gamma,$$

[13] The form of a taxation scheme in the model will be discussed in detail.

where K, L and α denote aggregate capital, aggregate effective labor, capital income share, respectively. Particularly, following Barro (1990), Baxter and King (1993), Fisher and Turnovsky (1995), and Leeper, Walker and Yang (2010), it is assumed that government spending, G, can affect a firm's production characterized by a parameter γ, which represents the degree of government expenditure externality or output elasticity of public expenditure. That is, an increase in public expenditure raises a firm's productivity directly. It is widely recognized that government spending on roads, public research spending, and medical services raises the potential production of an economy.[14] G can be interpreted as a stock of public capital as in Baxter and King and Leeper, Walker and Yang, while in this study it is considered as a flow of productive public services or public goods for production following Futagami, Iwaisako and Ohdoi (2008), Kamiguchi and Tamai (2011) and Albertini, Poirier and Roulleau-Pasdeloup (2014). However, the quantitative results of the two perspectives are similar.[15]

The representative firm makes decisions for labor and capital demand to maximize current profits such that:

$$\Pi_t = \max_{K_t, L_t} \left\{ K_t^{\alpha} L_t^{1-\alpha} G_t^{\gamma} - w_t L_t - (r_t + \delta) K_t \right\},$$

where δ is the depreciation rate for capital.

3.3.1.3 The Government

The government exogenously spends its expenditure in every period. Specifically, a government spending shock is assumed to follow a stochastic process with an AR(1) process:

$$g' = \rho_g g + \varepsilon_g, \quad \varepsilon_g \sim N(0, {\sigma_g}^2),$$

where g denotes log deviation of G from its steady state, i.e., $g \equiv \ln G - \ln G_{ss}$, and G_{ss} is the steady state level of government spending.[16]

[14]For empirical evidence on the productive government spending, see Bom and Ligthart (2014).
[15]Regarding this issue, see the working paper of Albertini, Poirier and Roulleau-Pasdeloup (2014).
[16]It is assumed that government spending shocks and the individual shocks are independent of each other.

The income tax schedule of an individual household is assumed to follow a log-linear tax function, which is characterized by two parameters λ and τ:

$$T(y) \equiv \begin{cases} \max\{(1-\lambda y^{-\tau})y, 0\} & if\ y \geq 0 \\ 0 & if\ y < 0 \end{cases}, \tag{3.1}$$

where y represents individual income, and τ denotes the progressivity of the tax system, and λ characterizes the average level of taxation.[17] $\max\{(1-\lambda y^{-\tau})y, 0\}$ implies no negative taxes or no public transfers. Suppose that y is positive. Positive (negative) τ means that the tax system is progressive (regressive): marginal tax rates are larger (lower) than average rates. Particularly, when $\tau = 0$, the tax system is affine taxation: marginal tax rates are the same as average rates, and, therefore, $T(y) = (1-\lambda)y$. This type of the taxation function is widely used in various studies such as Heathcote, Storesletten and Violante (2017) and Chang, Chang and Kim (2018).

3.3.1.4 Recursive Representation

Consider a recursive equilibrium for the model economy. For each household, the individual state variable is the vector $\theta \equiv (a, x)$, and the aggregate state is the vector $\Theta \equiv (\mu, G)$ where μ is a joint distribution of asset holdings and productivity across households. The Bellman equation for a employed worker $V^E(\theta, \Theta)$ is defined as:

$$V^E(\theta,\Theta) = \max_{c,a'} \left\{ \ln c - \chi \frac{\bar{h}^{1+1/\phi}}{1+1/\phi} + \beta \mathbb{E}\left[V(\theta', \Theta')\right] \right\}$$

subject to

$$c + a' = w(\Theta)x\bar{h} + (1+r(\Theta))a - T(w(\Theta)x\bar{h} + r(\Theta)a), c > 0,\ a' \geq \bar{b},$$

and

[17]With positive y, $1-\tau$ can be interpreted as a measure for the elasticity of after-tax income to before-tax income (Heathcote, Storesletten and Violante, 2017).

$$\mu' = \Psi(\mu, G)$$

where Ψ denotes a forecasting function for μ.

The value function for a non-employed worker, denoted by $V^N(\theta, \Theta)$, is:

$$V^N(\theta, \Theta) = \max_{c,a'} \{\ln c + \beta \mathbb{E}[V(\theta', \Theta')]\}$$

subject to

$$c + a' = (1 + r(\Theta))a - T(r(\Theta)a), c > 0,\ a' \geq \bar{b}, \text{ and } \mu' = \Psi(\mu, G)$$

The value function $V(\theta, \Theta)$ is defined as:

$$V(\theta, \Theta) = \max_{h \in \{0,\bar{h}\}} \left\{V^E(\theta, \Theta), V^N(\theta, \Theta)\right\}.$$

3.3.2 Definition of Equilibrium

A recursive competitive equilibrium is a transition operator $\Psi(\Theta)$, a set of factors $\{K(\Theta), L(\Theta)\}$, a set of value functions $\{V^E(\theta, \Theta)$,$V^N(\theta, \Theta)$, $V(\theta, \Theta)\}$, a set of market prices $\{w(\Theta), r(\Theta)\}$, and a set of policy functions $\{c(\theta, \Theta)$,$a'(\theta, \Theta)$,$h(\theta, \Theta)\}$ such that:

1. Individual optimization: Given market prices, $w(\Theta)$ and $r(\Theta)$, optimal decision rules $c(\theta, \Theta)$, $a'(\theta, \Theta)$ and $h(\theta, \Theta)$ solve the Bellman equations.

2. The firm's profit maximization: $K(\Theta)$ and $L(\Theta)$ satisfy $F_L(K, L, G) = w(\Theta)$ and $F_K(K, L, G) = r(\Theta) + \delta$ for all Θ.

3. Markets clearing: For all Θ,

   - Labor market clearing: $L(\Theta) = \int xh(\theta, \Theta)d\mu$,
   - Capital market clearing: $K(\Theta) = \int ad\mu$, and
   - Goods market clearing: $K(\Theta)^\alpha L(\Theta)^{1-\alpha} G^\gamma = C(\Theta) + I(\Theta) + G$ where $C(\Theta) = \int c(\theta, \Theta)d\mu$, and $I(\Theta) = K'(\Theta) - (1 - \delta)K(\Theta)$.

4. Balanced budget of the government: $G = \int T(w(\Theta)xh(\theta, \Theta) + r(\Theta)a)d\mu$ for all Θ.

5. Consistency of individual and aggregate behaviors.

Table 3.1: Parameters of the Model Economy

Parameter	Value	Description
β	0.98703	Time discount factor
$\overline{h}$	1/3	Extensive margin for hours worked
ϕ	0.3	Labor supply elasticity
χ	352.27	Parameter for disutility from working
ρ_x	0.939	Persistence of productivity shocks
σ_x	0.287	Standard deviation of productivity shocks
$\overline{b}$	-2.0	Borrowing constraint
α	0.33	Capital income share
δ	0.025	Capital depreciation rate
γ	0.15	Output elasticity of public expenditure
ρ_g	0.92	Persistence of government spending shocks
σ_g	0.012	Standard deviation of government spending shocks
G_{ss}/Y_{ss}	0.2	Steady state ratio of G to Y
λ	0.740	Parameter for average tax rate in steady state
τ	0.20	Progressivity of taxes in steady state
ω	0.85	Tax policy parameter

3.3.3 Calibration

In this subsection, I discuss calibration for the parameters used in the model economy. A simulation period is a quarter in the model. Table 4.1 summarizes the parameter values used in the model economy.

3.3.3.1 Preference and Borrowing Constraint

The parameter ϕ, which represents the micro elasticity of labor supply, is set to 0.3. This value is based on the findings that conventional micro estimates of the elasticity of labor supply are small $(0 - 0.5)$.[18] Fixed amount of hours worked, $\overline{h}$, is chosen to be 1/3. The time discount factor, β, and the disutility parameter of working, χ, are set so that quarterly return to capital is one percent, and the employment rate is 70 percent. The U.S. data such as the PSID and Survey of Consumer Finances (SCF) consistently report that employment

[18]It is noted that the choice of ϕ does not affect the simulated results due to indivisibility of a labor supply decision for households.

rates are around 70 percent.[19] The borrowing constraint, $\bar{b}$, is -2.0, which is approximately double the quarterly average income in the model economy.[20]

3.3.3.2 *Production Technology*

The capital income share, α, and the quarterly depreciation rate, δ, are calibrated to be 0.33 and 2.5 percent, respectively. The output elasticity of public capital, γ, differs substantially across studies. Bom and Ligthart (2014) estimate the average of the output elasticity of public expenditure over 578 different estimates from 68 studies for various countries. I compute the simple average of estimates for the U.S. from Table A1 of Bom and Ligthart and find that it is 0.148.[21] Thus, I set $\gamma = 0.15$.

3.3.3.3 *Labor Productivity*

For individual labor productivity shocks, previous studies in the literature including Floden and Linde (2001), French (2005), Chang and Kim (2006), and Chang, Kim and Schorfheide (2013) consistently report that the shocks are persistent and variance of them is also large. Following Chang, Kim and Schorfheide, I set $\rho_x = 0.939$ and $\sigma_x = 0.287$, which are estimated with the AR(1) wage process from the PSID.[22]

3.3.3.4 *Government*

Regarding the government spending shocks, I choose $\rho_g = 0.92$ and $\sigma_g = 0.015$ from the estimates of Zubairy (2014). The steady-state ratio of the government spending to GDP is set to be 0.2 ($G_{ss}/Y_{ss} = 0.2$). The steady-state level of tax progressivity, τ, is calibrated to be 0.2, which is the average of progressivity using two different measures for the sample

[19] Self-employed workers are included for the calculation of employment rates as they are also included in the CEX data.

[20] With this value, the fraction of households who own zero or negative wealth is around 20 percent, which is consistent with that in the U.S. data such as the PSID 1994.

[21] Bom and Ligthart (2014) find that the simple average estimate for the whole sample is 0.188.

[22] Chang, Kim and Schorfheide (2013) use the maximum-likelihood estimation (MLE) of Heckman (1979) when they estimate the AR(1) process of wage rates in logs to solve the self-selection problem for wage workers.

Table 3.2: Distribution of Wealth and Income

	Quintile					Gini
	1st	2nd	3rd	4th	5th	
			Panel A: Wealth Distribution			
Data						
SCF 1992	-0.39	1.74	5.72	13.43	79.49	0.78
PSID 1994	-1.22	0.88	4.98	14.68	80.68	0.79
Model Economy	-3.13	3.49	12.72	26.86	60.06	0.62
			Panel B: Income Distribution			
Data						
SCF 1992	2.18	6.63	11.80	19.47	59.91	0.57
PSID 1994	-0.27	5.06	13.94	24.80	56.48	0.57
Model Economy	1.59	6.13	13.15	24.08	55.05	0.53

Note: Distributions of wealth (Panel A) and income (Panel B) for U.S. data and the model economy. Statistics for the SCF 1992 are from Diaz-Gimenez, Quadrini and Rios-Rull (1997).

periods between 1980 and 2008.[23] In the steady state, the parameter λ is chosen so that the government runs a balanced budget.

3.4 Cross-sectional Distributions

As I investigate the distributional effects of the government spending shocks on consumption across the income quintiles, I first analyze if the model economy generates reasonable heterogeneity across individual households. Particularly, it is important to reasonably replicate consumption distribution or average consumption across the income quintiles as a means to an end for this study.

Table 3.2 summarizes the detailed information on income and asset holdings from the U.S data (the SCF 1992 and the PSID 1994) and the model.[24] Overall, the wealth is less

[23]Based on the same tax schedule function of this study, Heathcote, Storesletten and Violante (2017) find the similar value of the progressivity parameter.

[24]I use the PSID 1994 survey because this survey year has information on both wealth and income, and it falls in the midpoint of the sample period of the CEX in this study. For a robustness check, I use the SCF 1992, which is from Diaz-Gimenez, Quadrini and Rios-Rull (1997). Income is defined as all kinds of

Table 3.3: Characteristics of Income Distribution (ranked by income)

	Quintile					All
	1st	2nd	3rd	4th	5th	
Panel A: Data						
Share of income	-0.27	5.06	13.94	24.80	56.48	100
Share of after-tax/transfer income	4.87	9.18	14.92	24.06	46.94	100
Employment rate	17.51	67.90	85.82	93.76	96.25	72.2
Share of consumption	11.02	15.08	18.07	22.53	33.30	100
Average consumption ($\times\$10^3$)	1.21	1.66	1.99	2.48	3.66	2.22
MPC	0.58	0.49	0.47	0.44	0.40	0.48
Panel B: Model Economy						
Share of income	1.59	6.13	13.15	24.08	55.05	100
Share of after-tax/transfer income	2.21	8.27	15.65	25.36	48.51	100
Employment rate	10.79	50.92	88.76	99.82	99.99	70.0
Share of consumption	13.91	17.38	18.20	21.59	28.93	100
Average consumption ($\times\$10^3$)	1.54	1.93	2.02	2.40	3.22	2.22
MPC	0.86	0.63	0.60	0.80	0.56	0.69

Note: Characteristics of income distribution. Statistics for income and employment rates are from the PSID 1994. Statistics for after-tax/transfer income are from the PSID 1991. Statistics for consumption are computed from the CEX 1980:I-2008:III. The average quarterly per capita consumption for the data and the model is in thousands of U.S. dollars for year 2000. The values of MPC in the data are from Jappelli and Pistaferri (2014). The quarterly MPCs in the model are transformed into annual ones using the formula $1 - (1-\text{quarterly MPC})^4$.

concentrated in the model economy. The wealth Gini indexes for the SCF 1992 and the PSID 1994 are 0.78 and 0.79, respectively, while it is 0.62 in the model economy. However, the primary objective in this paper is not to account for the highly concentrated wealth distribution.[25]

The model economy successfully reproduces the income distribution, which is a baseline dimension of inequality in this paper to study the distributional effects on the government spending shocks on consumption across the income quintiles. In both data and model economy, households in the first quintile earn less than three percent of total income, and the

revenue for family or households before taxes and transfers, and wealth is defined as the net worth of family or households.

[25]For studies on highly concentrated wealth distribution in the context of the heterogeneous agent model frameworks, see Quadrini (2000), Castaneda, Diaz-Giminez and Rios-Rull (2003) and Cagetti and De Nardi (2006).

income share for the fifth quintile is around 60 percent.[26] The Gini coefficient for income in the model is 0.53, which is close to the data (0.57). In order to take a close look at characteristics of the income distribution, I compute various dimensions of inequality, including the consumption shares and average consumption (per capita) across the income quintiles for the data and the model economy in Table 3.3.[27] The model economy reasonably replicates the observed after-tax/transfer income distribution: the share of after-tax/transfer income for the poorest increases compared to that of before-tax/transfer income while it decreases for the richest. Also, as shown in the U.S data, the increasing pattern of labor supply (the employment rates) across the income quintiles is well-reproduced in the model economy.[28] Next, the consumption shares across the income distribution are successfully reproduced: households in the lowest income quintile consume 11.02 percent and 13.91 percent of total consumption in the data and the model, respectively, while households in the top quintile consume 33.3 percent and 28.93 percent of total consumption in the data and the model, respectively. Importantly, average consumption (per capita) across the income quintiles in the model economy fits the U.S. data well.[29] The first and the fifth quintiles of the income distribution in the model economy consume \$1,540 (in U.S. dollars for year 2000) and \$3,220 over a quarter, respectively, which are comparable to the data (\$1,210 and \$3,660, respectively).

Next, I compare the marginal propensity to consume (MPC) produced in the model to the empirical values in the literature. The average MPC in the model is 0.69, which is

[26]Since income is defined as before-tax/transfer income, it can be negative. The negative income for the income-poorest may come from their business or capital losses (Diaz-Gimnnez, Glover and Rios-Rull, 2011). Using the SCF, Diaz-Gimenez, Quadrini and Rios-Rull (1997) and Diaz-Gimnnez, Glover and Rios-Rull (2011) also find that the income-poorest have negative income. Note that the share of after-tax/transfer income for the poorest is positive.

[27]Statistics for income and employment rates are from the PSID 1994, and information on consumption is computed from the CEX averages. Statistics for after-tax/transfer income are from the PSID 1991 because the PSID has not provided estimates of federal income tax payments for each tax unit since 1992.

[28]The model generates the increasing pattern of labor supply by income quintiles thanks to a progressive tax function, but this is typically hard to obtain in a standard incomplete market model.

[29]For comparison between the data and the model economy, I adjust the units of consumption in a way that average consumption of the model economy matches mean consumption in the CEX. Average quarterly consumption for data and the model is in U.S. dollars for year 2000.

much larger than the 0.33 reported in Sahm, Shapiro and Slemrod (2010) or the 0.48 in Jappelli and Pistaferri (2014).[30] The model economy generates the higher MPC since both individual and aggregate shocks are highly persistent, while most of the estimates of MPC in the existing empirical papers are based on the tax rebate policies in 2001 and 2008 (e.g., Johnson, Parker and Souleles (2006) and Sahm, Shapiro and Slemrod (2010) among others), which are temporary changes in income.[31] The MPCs across income quintiles for the data and the model are reported in the last row of Panel A and B in Table 3.3, respectively. As reported in Jappelli and Pistaferri, there is a wide range of heterogeneity in the MPC across households, and it declines with income.[32] The model economy also broadly replicates the decreasing pattern of the MPCs across income groups even if the MPC in the fourth quintile jumps.[33] Following Chang and Park (2017), I also focus on the MPCs across income groups relative to the average since a direct comparison between the levels of MPC for the model and the empirical estimates may not be fair. The relative MPCs for the first and fifth income quintile in the model are 1.25 and 0.82, respectively, while the corresponding empirical values in the data are 1.22 and 0.83, respectively. Thus, the model generates MPCs that are quite similar to those in the data in a relative sense.

From the results in this section, I argue that the model economy generates reasonable heterogeneity to study the distributional effects of the government expenditure on consumption over income levels.

[30]The model-implied MPC is obtained using a simple regression: I regress consumption on after-tax income with constant term based on 1,000 periods of quarterly time series for each income group. I transform the quarterly into an annual MPC using the formula $1-(1-\text{quarterly MPC})^4$.

[31]Similarly, a Huggett-style incomplete market model by Chang and Park (2017) also generates the higher average MPC of 0.85.

[32]Jappelli and Pistaferri (2014) use the 2010 Italian Survey of Household Income and Wealth when estimating the MPCs.

[33]Using a life-cycle model with liquid and illiquid assets, Kaplan and Violante (2014) show a striking difference of MPCs out of the fiscal stimulus payment for those who are hand-to-mouth and those who are not. A recent paper by Chang and Park (2017) reports the comparison of MPC's between the quantitative model and the data.

3.5 Distributional Effects of Government Spending

3.5.1 Taxation scheme

In this section, I investigate the effects on the government spending shocks, focusing on the responses of consumption across the income quintiles. For the taxation scheme, I assume that government expenditure is partially financed by adjusting progressivity. Specifically, given the tax function previously discussed, a path of τ and λ will be adjusted so that the government keeps a balanced budget when government spending unexpectedly increases. Since there are infinitely many combinations of the two parameters which satisfy the balanced government budget, I assume that the government employs a simple linear rule for the combination of the two parameters such that:

$$(1-\omega)\widetilde{\tau} + \omega\widetilde{\lambda} = 0, \tag{3.2}$$

where $\widetilde{z}$ denotes an absolute deviation of a variable, z, from its steady state. Notice that the parameter ω captures weight on the policy only financed by progressivity changes. For example, $\omega = 1$ means that government spending is only financed by changing τ with fixed λ.

The next question is how government spending affects tax progressivity. To see this, I compute a measure for tax progressivity using the log-linear function for the tax system in Equation 3.1. Suppose that $y > 0$. Following Heathcote, Storesletten and Violante (2017) and Ferriere and Navarro (2017), tax progressivity, τ, can be computed such that:

$$\tau = \frac{T'(y) - T(y)/y}{1 - T(y)/y}. \tag{3.3}$$

According to Equation 3.3, measures for the average tax rate ($T(y)/y$) and the average

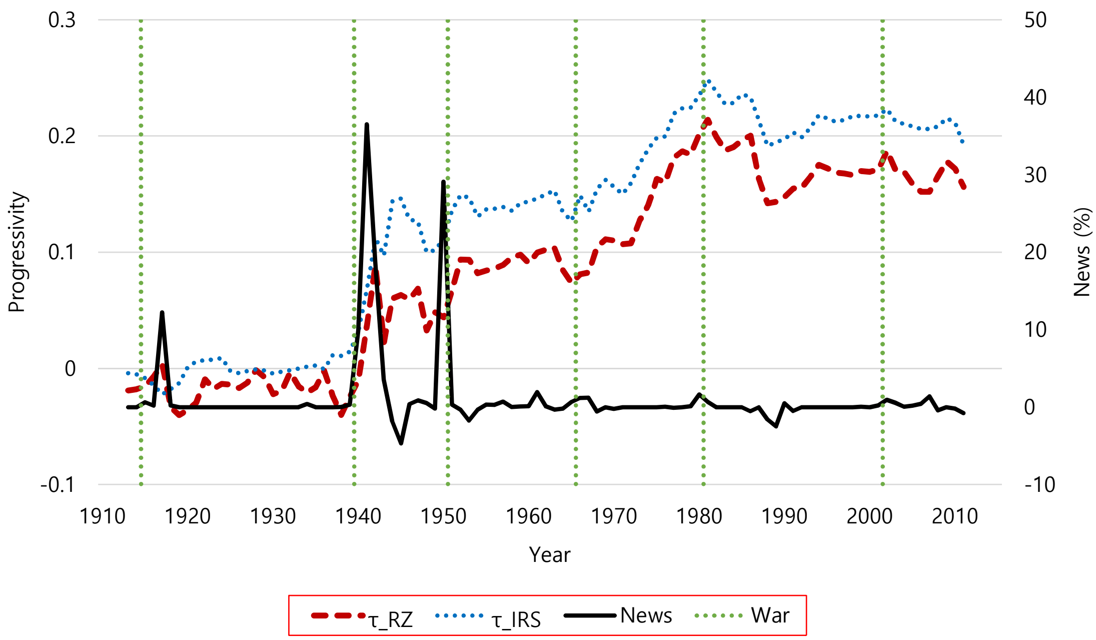


Figure 3.2: Historical Trends of Tax Progressivity

Note: Historical trends of tax progressivity with two different measures. Measures for tax progressivity is computed using Equation 3.3. Given the average marginal tax rates which are from Barro and Redlick (2011) and Mertens and Montiel Olea (2018), τ_RZ is measured using information on nominal tax and nominal GDP from Ramey and Zubairy (2018) while τ_IRS is computed using information on taxes and total income from IRS. News variables are defined as the fraction of nominal GDP of the previous year (%). War periods are WWI (1914), WWII (1939), the Korean War (1950), Vietnam War (1965), Soviet Invasion to Afghanistan (1980), and 9/11 (2001).

marginal tax rate ($T'(y)$) across income levels are needed to obtain a measure for progressivity, τ. I use information on the share of nominal tax over nominal GDP from Ramey and Zubairy (2018) as a measure for the average tax rates. As a robustness check, I use the time series for total taxes (total tax liability) and income from Internal Revenue Service (IRS). For the average marginal tax rate ($T'(y)$), the historical data computed by Barro and Redlick (2011) and Mertens and Montiel Olea (2018) are used.[34] In particular, as far as the measures for government spending shocks are concerned, I use the defense news series built by Ramey and Zubairy. News variables are defined as the fraction of nominal GDP of the previous year. Figure 3.2 reports the historical trends of measures for U.S tax progressivity, news shocks, and the years the wars started over the period of 1913-2011.[35] An interesting

[34]Data for the periods of 1913-1945 are from Barro and Redlick (2011), and time series for 1946-2011 are from Mertens and Montiel Olea (2018).

[35]War periods are WWI (1914), WWII (1939), Korean War (1950), Vietnam War (1965), Soviet Invasion to Afghanistan (1980), and 9/11 (2001).

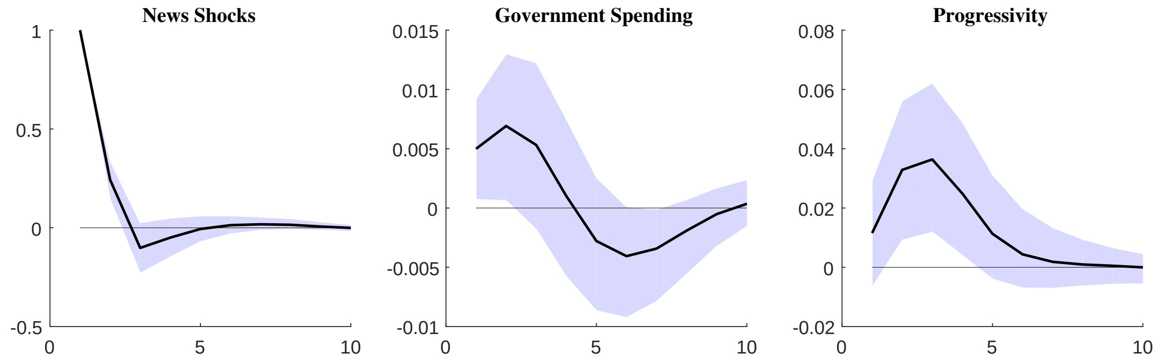


Figure 3.3: Responses of Tax Progressivity to News Shocks

Note: Responses of real per capita government spending and tax progressivitiy (τ_RZ) to news shocks for the sample period 1950-2008. The government spending and tax progressivitiy are logged. The shaded regions are the 68 percent confidence bands generated by Monte Carlo simulations. The horizontal axis reports the number of years after the shock.

fact emerges from Figure 3.2: progressivity of taxes tends to rise with positive news shocks. For example, tax progressivity skyrocketed during war time such as WWI (1914), WWII (1939), the Korean War (1950) and, more recently, 9/11 (2001).

In order to quantitatively investigate whether and how much progressivity increases with an unexpected rise in government spending, I estimate response functions of progressivity to defense news shocks based on the VAR where the defense news shocks are ordered first, and constant terms, quadratic trend terms, and a one-period lag are included. I use an annual sample for the post-WWII periods but excluding the ZLB periods and onward to be consistent with the CEX sample (1980:I-2008:III). As shown in Figure 3.3, tax progressivity rises in response to positive public spending shocks. These results are in support of evidence that a rise in government spending is accompanied by an increase in tax progressivity. Table 3.4 reports progressivity elasticities of government spending across different samples and estimation approaches. According to Table 3.4, the progressivity elasticity of government spending is around 4 - 6. Hence, I choose $\omega = 0.85$ so that the elasticity is around five.[36] This finding is in line with work of Vélez (2014) and Ferriere and Navarro (2017). Vélez shows that progressive income taxation in the twentieth century is a byproduct of war using

[36]Ferriere and Navarro (2017) use a much larger value for the progressivity elasticity of government spending: they assume that the elasticity is 10 on impact for the "Higher Progressivity" case.

Table 3.4: Progressivity Elasticity of Government Spending

Peak	Two-year Integral	Four-year Integral
5.26	3.73	5.80

Note: Progressivity elasticity of government spending for the sample period 1950-2008.

a long time series of the top marginal personal income tax rate of multiple countries. Ferriere and Navarro also find that the effect of government spending on macro variables is much larger when it is financed with more progressive taxes. Similarly, Barro and Redlick (2011) and Mertens and Montiel Olea (2018) report that significant increases in the federal average marginal income-tax rate are related to wartime.

3.5.2 Results

3.5.2.1 Responses of Aggregate Variables

The responses of key aggregate variables in the model economy to government spending shocks for 20 quarters of horizon are shown in Figure 4.2. In response to a positive government spending shock, output increases significantly. The impact or peak output multiplier for government expenditure in the model economy is around one, which is comparable to empirical findings of Blanchard and Perotti (2002), Fatas and Mihov (2001), and Zubairy (2010). Particularly, the model economy generates the fact that consumption and the wage rate respond positively to government spending shocks, while the standard RBC model predicts a reduction in wages and consumption.[37] The peak consumption multiplier for government expenditure in the model economy is around 0.03.[38] Investment is crowded out by public spending shocks, which is also found in empirical work such as Blanchard and Perotti, Mountford and Uhlig (2009), and Zubairy. Lastly, hours (employment) and interest rates also rise significantly as in empirical analyses in the literature.

[37]A rise in consumption and wages in response to a positive government spending shock is supported by most of the empirical studies in the literature, such as Fatas and Mihov (2001), Blanchard and Perotti (2002), Gali, Lopez-Salido and Valles (2007), Zubairy (2010), and Ramey (2011).

[38]See Blanchard and Perotti (2002), Gali, Lopez-Salido and Valles (2007), Mountford and Uhlig (2009) and Ramey (2011) for the estimates of consumption multipliers of government spending.

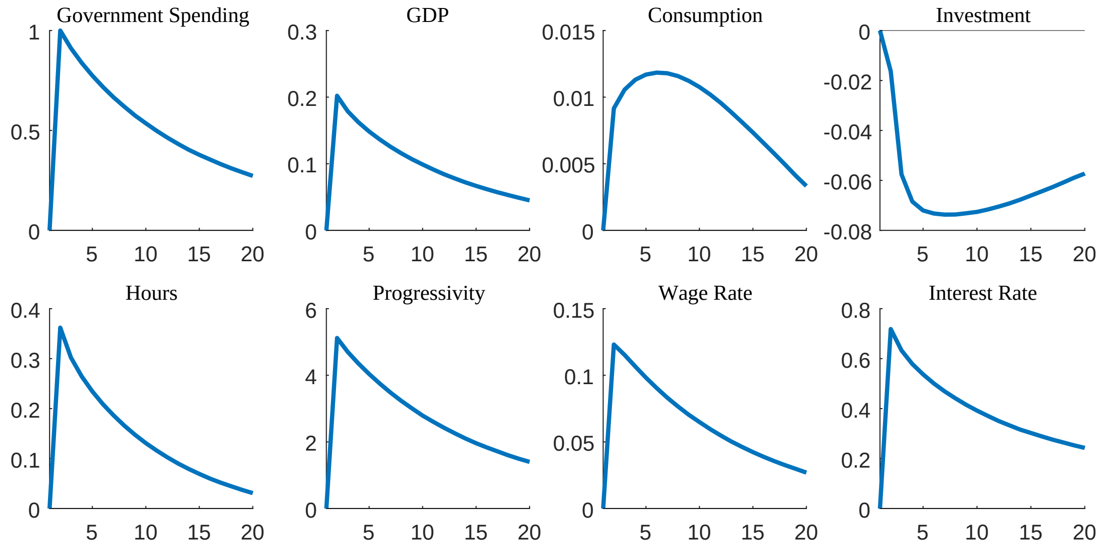


Figure 3.4: Impulse-responses of Aggregate Variables: Model Economy

Note: Responses of aggregate variables to a government spending shock. All variables other than interest rates are logged.

3.5.2.2 Distributional Effect on Consumption

The main focus of this study is on the distributional effects of government spending shocks on consumption across the income distribution. Figure 3.5 exhibits the responses of consumption across the income quintiles to a positive government spending shock. The model economy successfully matches the responses of consumption between the poor and the rich with the empirical findings discussed earlier. According to Figure 3.5, consumption of households in the first three income quintiles increases while that of richer households (the top two quintiles) decreases. Table 3.5, which reports consumption multipliers for government expenditure across income groups, confirms these findings. The peak multipliers are positive for the poor income groups and negative for the rich. For example, the peak multiplier of the first income quintile is 0.28 while that of the fifth quintile is -0.69. The signs of one-year and two-year integral multipliers are also well replicated.

The intuitive explanation for the results is as follows. Under the assumption of the divisible labor decision, as discussed in Hall (2009), it is the intratemporal optimality condition – jointly determining optimal hours and consumption – that has a key mechanism accounting

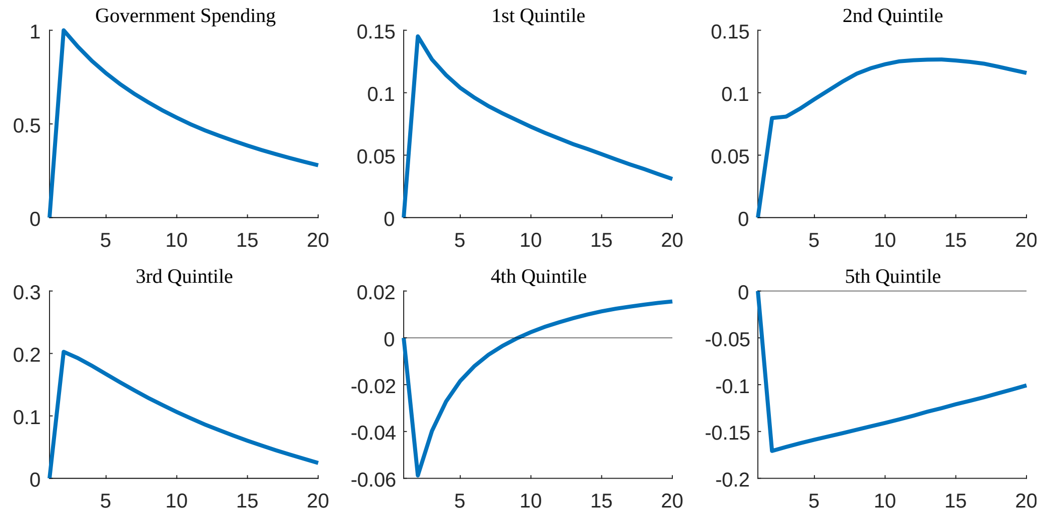


Figure 3.5: Responses of Consumption across Income Quintiles

Note: Responses of average per capita consumption across the income quintiles to a government spending shock. All variables are logged. "1st Quintile" denotes the lowest income quintile, and "5th Quintile" denotes the top income quintile.

Table 3.5: Multipliers across Income Quintiles: Model Economy

Variable	Peak IRF	1 Year Integral	2 Year Integral
1st Quintile	0.28	0.27	0.27
2nd Quintile	0.31	0.24	0.32
3rd Quintile	0.52	0.54	0.54
4th Quintile	-0.18	-0.12	-0.08
5th Quintile	-0.69	-0.76	-0.84

Note: Consumption multipliers across the income quintiles in the model economy. "1st Quintile" denotes the lowest income quintile, and "5th Quintile" denotes the top income quintile.

for the effect of government expenditure on consumption. Hence, after-tax wage rates are crucial to determine the response of consumption to a spending shock. If government spending is financed by changing progressivity, a substantial degree of heterogeneity of post-tax wage rates across the income distribution arises. Therefore, the tax system associated with the spending policy plays a crucial role in the different responses of consumption between the poor and the rich. When the government increases its spending unexpectedly, there are two main effects on after-tax wage rates in the model economy: a rise in tax progressivity to finance the rise in spending and an increase in labor demand due to productive public expenditure. The former differently affects post-tax wages across income levels while the latter is an economy-wide effect on after-tax wage rates. Specifically, with positive income, the after-tax wage rate, w_{at}, is given by

$$w_{at} \equiv \lambda\{wxh + ra\}^{-\tau} w. \tag{3.4}$$

Suppose that λ is fixed for simplicity. Then, for given prices, r and w, a rise in τ after a spending shock allows changes in w_{at} to be different across income levels: w_{at} tends to increase for the poor but decrease for the rich. Next, productive government spending increases labor demand and in turn raises wage rates, w.[39] An interaction between productive government spending and a tax progressivity change generates heterogeneous responses of the after-wage rates across households. w_{at} increases significantly for consumers in the lower income quintiles since an increase in wage rates induced by productive government spending dominates a slight rise in tax rates. In contrast, w_{at} falls for those who are in the top quintiles because the effect of productive government spending is dominated by the effect of tax progressivity. Panel B in Figure 3.6, which depicts responses of w_{at} across the income quintiles to a positive government spending shock, supports this mechanism: the first four income groups show the positive responsiveness of w_{at} while it significantly decreases for the

[39]Of course, an increase in the prices results in a reduction in w_{at}, but this effect may not be large with small positive τ.

Table 3.6: Marginal Worker Distribution (ranked by income)

	Quintile					All
	1st	2nd	3rd	4th	5th	
$\xi = 0.03$	11.96	7.45	6.77	3.53	0.53	6.05
$\xi = 0.05$	18.49	12.54	10.67	5.44	0.82	9.59
$\xi = 0.07$	24.34	17.72	14.73	7.56	1.15	13.10

Note: Shares of marginal workers across the income quintiles. Marginal workers are defined as Equation 3.5. "1st Quintile" denotes the lowest income quintile, and "5th Quintile" denotes the top income quintile.

top income quintile.

When a labor supply decision is indivisible, post-tax wage rates are much more important for the consumption dynamics since labor supply elasticity for the poor becomes even larger with the indivisibility of labor supply. Table 3.6, which shows shares of marginal workers across the income quintiles, provides evidence for it. A marginal worker is defined as a worker who is almost indifferent between working and not-working at the steady state. Formally, given assets holding, a, and productivity, x, a household is a marginal worker when $m(a,x) = 1$, and $m(a,x)$ is given by:

$$m(a,x) \equiv \begin{cases} 1 & if \ \left|V^E(a,x) - V^N(a,x)\right| < \xi \\ 0 & otherwise \end{cases}, \tag{3.5}$$

where ξ is a small real number. According to Table 3.6, marginal workers are relatively many at the bottom of the income distribution. For example, when $\xi = 0.05$, around 20 percent of households are marginal workers in the first income quintile, while the share of marginal workers is less than one percent in the top quintile.[40] In other words, labor supply elasticity of lower-income consumers is much larger than that of rich households. Accordingly,

[40] I choose $\xi = 0.05$ as a benchmark number since the share of workers who change their employment status is 9.6 percent in the steady state.

non-employed households (before a spending shock) in the lower income groups, who are likely to be marginal workers as shown in Table 3.6, become employed after the shock since w_{at} is greater than the reservation wage rates. Figure 3.6 confirms this story line: households in the first three lowest income quintiles increase hours of work or decide to be employed due to a significant rise in w_{at}.[41] In turn, this allows the poor to increase after-tax income and consumption as found in Figure 3.6 and Figure 3.5. However, as shown in Figure 3.6, hours worked for consumers in the upper quintiles (the fourth and the fifth income groups) drop, and their post-tax income remains almost constant or falls since w_{at} increases a bit or decreases.[42] Finally, the rich reduce their consumption (Figure 3.5). If λ is allowed to change following the tax policy rule in Equation 3.2, the main results are still the same.[43] Interestingly, the behavior of consumers in the fourth income group is different from the others. For the poor (the first three quintiles), the direction of hours is consistent with that of w_{at}, which implies that the substitution effect is larger than the income effect. The top income quintile does not change their hours of work since w_{at} for a majority of the richest is still greater than the reservation wage rates, or the share of marginal workers is very small. However, households in the fourth income group decrease their consumption and hours with a small rise in w_{at}, which suggests that the income effect dominates the substitution effect.

I also provide supportive empirical evidence for the different responses of employment across income groups to government spending shocks, which is the key channel of the model. For this analysis, I use the Current Population Survey (CPS) since the CPS has better information on labor force participation than the CEX. Since information on income in the CPS is available in March of every year, I cannot use quarterly data for employment rates across income quintiles. Hence, I use annual data spanning 1980 to 2014[44] and use the SPF

[41] Of course, employed workers at the bottom quintiles of the income distribution also can increase their consumption due to an increase in after-tax income induced by a rise in w_{at} for them.

[42] Not surprisingly, there might be households in the richest quintile who lose their jobs but this effect may not be large since most of them are not marginal workers as in Table 3.6.

[43] Compared to fixed λ, w_{at} for the poor (for the rich) may increase (decrease) less since τ increases less.

[44] Due to the small sample size, I do not exclude the ZLB period for this analysis.

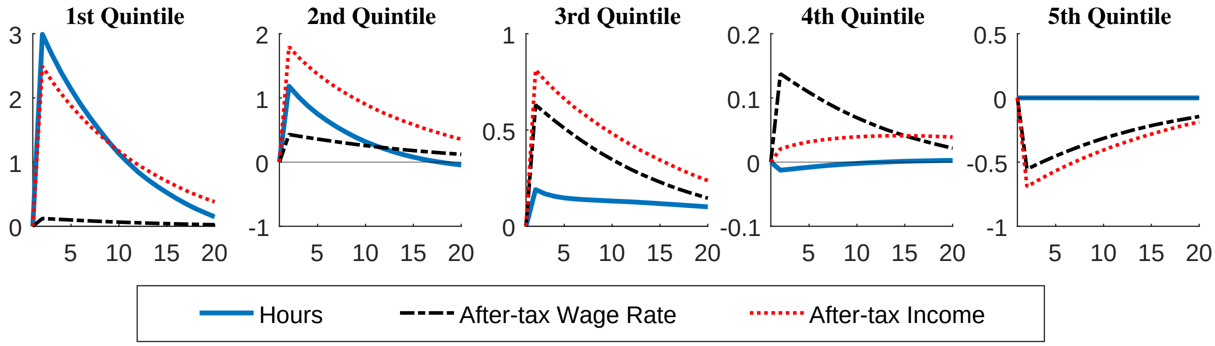


Figure 3.6: Responses of Hours, After-tax Wage Rates, and After-tax Income across Income Quintiles

Note: Responses of average hours , average after-tax wage rates, average after-tax income across the income quintiles to a government spending shock. All variables are logged. "1st Quintile" denotes the lowest income quintile, and "5th Quintile" denotes the top income quintile.

forecast errors as the measure of government spending shocks.[45] To estimate the effects of a government spending shock on employment by income, I employ the VAR where the SPF forecast errors are ordered first, and constant terms, quadratic trend terms, and a one-period lag are included. Figure 3.7 reports the estimated response of employment rates across income groups. As predicted by the model economy, households in the lower income quintiles increase employment while employment rates for richer households tend to decrease in response to positive government spending shocks. This supports the model prediction of heterogeneous effects of government spending on employment between the poor and the rich.

3.5.2.3 Role of Indivisible Labor

Indivisibility of labor choice allows for producing heterogeneous Frisch elasticity of the labor supply across income levels since individuals have different reservation wages: labor supply elasticity in the bottom of the income distribution is very large while that of rich households is almost zero.[46] In contrast, the Frisch elasticity is the same between the poor and the rich under the assumption of a flexible hours decision. A discrete choice for labor

[45] In this case, the measure of SPF forecast errors is defined as the difference between the actual federal spending growth from a year ago in March and the four-quarter-ahead SPF forecast of government spending in March.

[46] The Frisch labor supply elasticity is defined as the percentage change in hours caused by one percentage change in wages, abstracting from the effect on wealth.

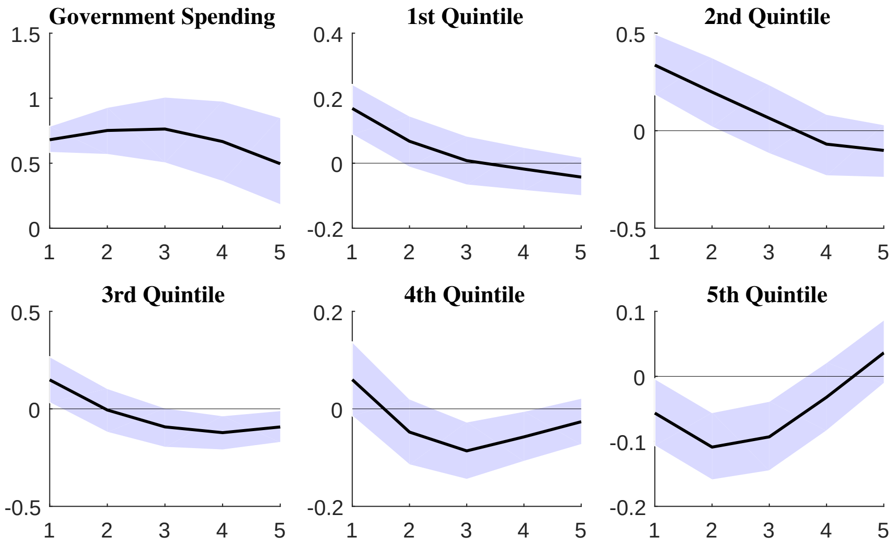


Figure 3.7: Responses of Employment Rate across Income Quintiles

Note: Responses of employment rates across the income quintiles to a government spending shock based on the CPS 1980-2014. "1st Quintile" denotes the lowest income quintile, and "5th Quintile" denotes the top income quintile.

supply plays two important roles in the model economy. First, the assumption allows the government to increase progressivity to finance a rise in government spending since labor supply elasticity of higher-income households is very small under the assumption of indivisible labor. Second, households in the lower income groups can significantly increase hours worked with indivisibility of a labor choice, which allows the poor to increase consumption. However, the above two facts are incompatible in a model with divisible labor. On the one hand, if elasticity of the labor supply is too high in the divisible labor model, the government should decrease progressivity to increase tax revenue, which is inconsistent with the empirical evidence. On the other hand, when the labor supply is very inelastic, the poor cannot increase their consumption due to a small rise in hours. The latter case is reported in Figure 3.8.

Figure 3.8 compares the responses of key aggregate variables (Panel A) and consumption across the income quintiles (Panel B) to a government spending shock in the indivisible and divisible labor models. For the divisible labor model economy (DL model), I set the same value of Frisch labor elasticity (0.3) as in the indivisible labor model (IL model).[47] As

[47]For other parameter values in the DL model, the same calibration strategies used in the IL model are applied.

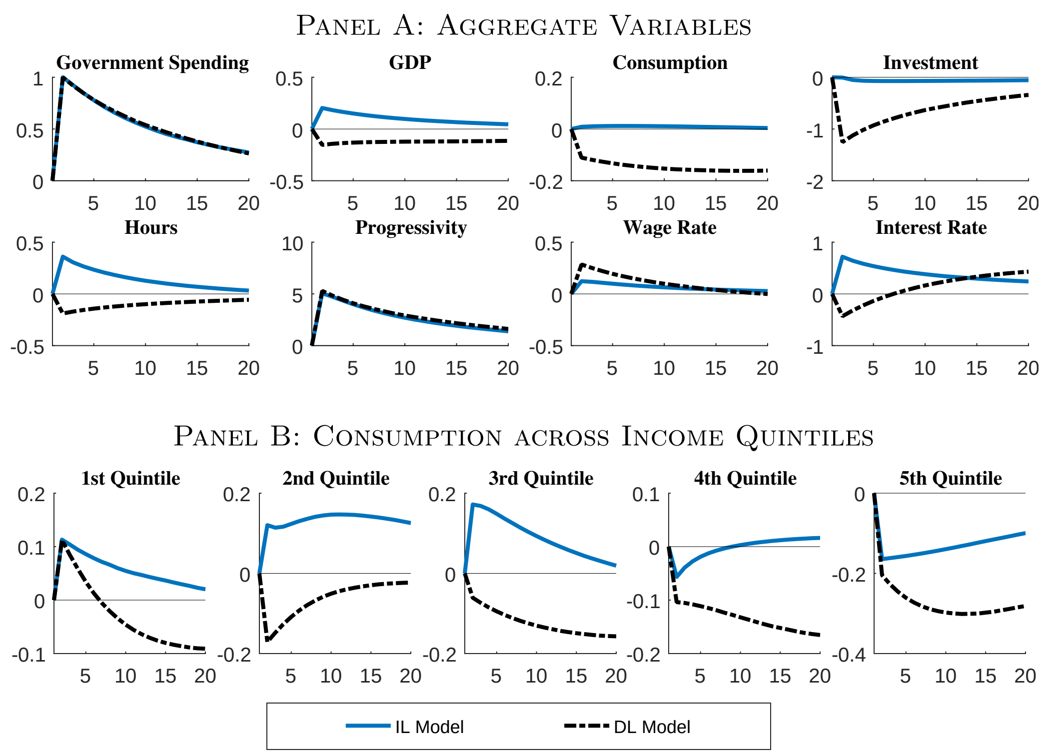


Figure 3.8: Impulse-responses with Indivisible and Divisible Labor

Note: Responses of aggregate variables (Panel A) and consumption across the income quintiles (Panel B). "IL" and "DL" denote indivisible and divisible labor, respectively. All variables other than interest rates are logged. "1st Quintile" denotes the lowest income quintile, and "5th Quintile" denotes the top income quintile.

found in Figure 3.8, the DL model fails to replicate the responses of both macro and micro variables even if progressivity increases as much as in the IL model. The small elasticity of the labor supply for poor households results in a decrease in consumption for most of them. In contrast, since the labor supply elasticity for the rich in the DL model is relatively large compared to that in the IL model, they reduce their hours as a consequence of a decrease in after-tax wage rates, which leads to a drop in aggregate hours. The relatively large labor supply elasticity for higher income households also results in a substantial decrease in income and causes their investment and consumption to drop significantly. Consequently, aggregate consumption and investment fall, and a reduction in hours finally decreases output in the DL model.

3.5.2.4 Role of Productive Government Spending and Taxation Scheme

The interaction between productive government expenditure and the more progressive taxation scheme in response to an unexpected rise in government spending generates the heterogeneous effects of government spending on consumption by income as well as the reasonable responses of the key aggregate variables. In order to investigate the marginal contributions of each, I consider three additional model economies: an economy with productive government spending only ($\gamma = 0.15$, $\tau = 0$), an economy with progressive taxation only ($\gamma = 0$, $\tau = 0.2$), and an economy with neither of them ($\gamma = 0$, $\tau = 0$). Except for γ and τ, calibration strategies for the three models are the same as those used for the benchmark model economy.

Figure 3.9 shows the responses of the key macro variables (Panel A) and consumption across income groups (Panel B). Productive government spending alone does not generate the observed aggregate effects of government spending shocks. When $\tau = 0$, the tax system is linear. This implies that changes in tax rates in response to government spending shocks are the same for all the households. In the model with productive government spending only, a rise in after-tax wage rates due to productive government spending is dominated by the rise in tax rates. Accordingly, both aggregate hours and consumption decrease after positive

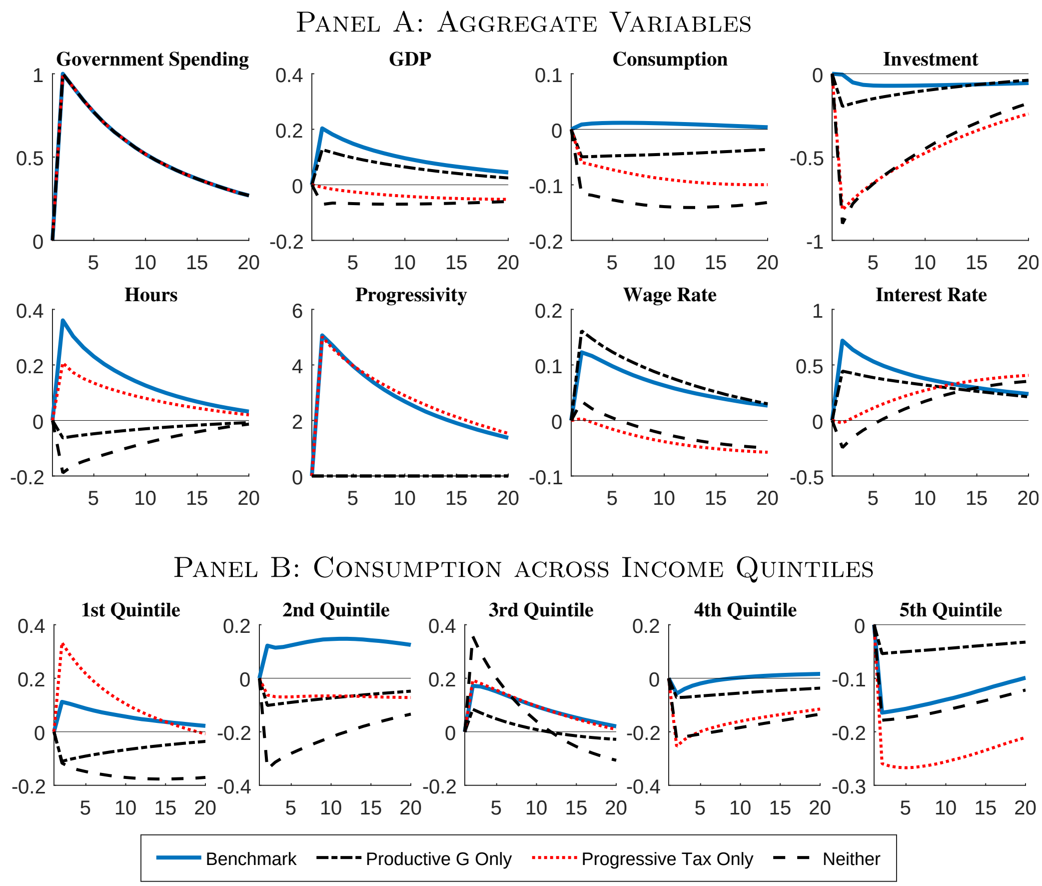


Figure 3.9: Impulse-responses for the Four Models

Note: Responses of aggregate variables (Panel A) and consumption across the income quintiles (Panel B). All variables other than interest rates are logged. "1st Quintile" denotes the lowest income quintile, and "5th Quintile" denotes the top income quintile. "Productive G Only" denotes an economy with productive government spending only, and "Progressive Tax Only" refers to an economy with progressive tax only, and "Neither" denotes an economy with neither of them.

government spending shocks. Productive government spending only also fails to explain the heterogeneous responses of consumption across income groups since there are no different effects on after-tax wage rates across households under the affine taxation: as found in Panel B of Figure 3.9, households in all the income quintiles other than the third quintile reduce their consumption.[48]

Next, the model economy with progressive taxation only shows that the progressive tax system alone cannot account for the responses of the key macroeconomic variables even though it replicates the distributional effects on consumption between the poor and the rich.[49] On aggregate, output and wages decrease in this model in response to an unexpected rise in public spending, which is not comparable to the empirical findings in the literature. In this case, firms do not increase labor demand since the model excludes productive government spending. Hence, hours increase by less compared to the benchmark model, and wage rates decrease. In the model economy with progressive taxation only, there are more heterogeneous effects on after-tax wages since an economy-wide effect on after-tax wage rates induced by productive government spending is omitted.[50] Thus, rich households tend to reduce investment and consumption considerably while households in the bottom of the income distribution can increase their consumption. Finally, aggregate investment and consumption fall significantly mainly due to a substantial decrease in after-tax income from the rich, and it in turn leads to a reduction in output.

By the same logic, the model economy with neither productive government spending nor a progressive tax system fails to account for the responses of both aggregate and disaggregate

[48] Compositional changes may cause consumption in the third income quintile to increase.

[49] The reason why consumption in the second quintile shows a negative sign in the "Progressive Tax Only" model is as follows. In the model economy, households in the second income group own relatively large amounts of wealth due to the assumption of indivisible labor. Hence, the response of post-tax interest rates to government spending shocks relatively matters for them. With productive government spending, the second quintile can increase their consumption and investment at the same time since after-tax interest rates rise significantly. However, without productive government spending or in the model of the "Progressive Tax Only", an increase in after-tax interest rates is relatively small. Therefore, they want to save more and consume less since the intertemporal substitution effect is larger than the income effect.

[50] In the model economy with progressive taxation only, ω is chosen to be 0.97 which is larger than the 0.85 in the benchmark model. Hence, the larger ω causes the model economy to have more heterogeneous effects on post-tax wage rates across households.

variables to a spending shock. From this analysis, I can conclude that the interaction between productive government spending and a progressive taxation scheme plays a crucial role in explaining the distributional effects of government spending on consumption by income as well as the responses of the key aggregate variables.

3.6 Conclusion

This paper tries to uncover why consumers behave differently in response to a government spending shock. To this end, I construct a heterogeneous agent model economy which incorporates a progressive taxation scheme, productive government expenditure, and indivisible labor. I find that the model economy successfully replicates the different responses of consumption between the bottom and the top of income distribution to government spending shocks. When the government increases its spending accompanied with a rise in tax progressivity, poor households are employed and hence increase their consumption due to an increase in after-tax wage rates while the rich decrease consumption since the effect of productive government spending cannot fully offset a significant increase in tax rates.

Existing theoretical macroeconomic models inspired by Gali, Lopez-Salido and Valles (2007) suggest that credit-constrained consumers are crucial to account for why government spending shocks have substantially different effects on consumers. On the other hand, this study proposes a new perspective by suggesting that it is important to consider different tax burdens across consumers when studying the distributional effects of government spending shocks.

4. MONETARY POLICY AND INEQUALITY: HOW DOES ONE AFFECT THE OTHER?

4.1 Introduction

The unequal distributions of income and wealth have become a primary concern for economists and policy makers (Piketty, 2014; White House, 2017). In recent years, such inequality is particularly relevant for the stabilization role of economic policies since policies might have disparate effects across different segments of the population. This study investigates the relation between monetary policy and inequality. On the one hand, discretionary changes in monetary policy actions can have distributional effects on consumption, income, and wealth in the short run.[1] On the other hand, the level of inequality may affect the effectiveness of monetary policy in the long run.[2] Specifically, this paper studies both *the effects of monetary policy on inequality* and *the role of the long-run level of inequality in the effectiveness of monetary policy*, by answering the following questions:

1. How does discretionary monetary policy affect income, consumption and wealth inequality, and what is the key mechanism?
2. How does the long-run level of inequality affect the effectiveness of monetary policy in terms of output, and what is the linkage?

In regards to the first question, an increasing concern about economic inequality has resulted in monetary policymakers discussing the potential distributional effects of monetary policy instruments (Mersch, 2014; Bullard, 2014; Bernanke, 2015). However, how monetary policy actions affect inequality is still ambiguous since there are a number of underlying channels as pointed out by Mersch (2014) and Bernanke (2015).[3] In this paper, I propose a new

[1]In this paper, I do not focus on the effect of systematic parts of monetary policy.

[2]The "effectiveness of monetary policy" is defined as the extent to which monetary policy affects output, i.e., the responsiveness of output.

[3]Coibion et al. (2017) discuss the possible channels of the distributional effects of monetary policy: portfolio channel (households hold different financial assets), the savings redistribution channel (savers and

transmission mechanism of the distributional effects of monetary policy by developing a heterogeneous agent model economy with nominal wage contracts and indivisible labor.[4] In this model economy, nominal wage contracts are important for explaining the disaggregate effects of monetary policy shocks. As far as the second question is concerned, how the long-run level of economic inequality affects the effectiveness of monetary policy is also an important issue, when considering different degrees of inequality across countries and even within a country. In spite of its importance, this topic is relatively unstudied in the literature, and existing papers (e.g., Kim (2017) and Cravino, Lan and Levchenko (2018)) do not have a general consensus.[5] Filling this gap is another primary goal of this study.

To study the relation between monetary policy and inequality, I build a dynamic stochastic general equilibrium (DSGE) model where there is a large population of heterogeneous households, a government, and many identical firms. In the model economy, it is assumed that asset markets are incomplete as in Huggett (1993) and Aiyagari (1994) in that households cannot fully insure against their idiosyncratic productivity shocks. Cash-in-advance constraints are considered in the standard real business cycle model as in Cooley and Hansen (1989).[6] Importantly, nominal wage contracts are also introduced in the model economy as in Cho and Cooley (1995) and Janko (2008). Nominal wage contracts play an important role in

borrowers are differentially affected by monetary policy), the financial segmentation channel (households have different access to financial markets and instruments). Auclert (2017) focuses on the three channels: an earnings heterogeneity channel from unequal income gains, Fisher channel from unexpected inflation, and interest rate exposure channel from real interest rate changes.

[4]Some recent studies in the literature provide channels of redistributive effects of monetary policy in the context of New Keynesian frameworks. For example, Kaplan, Moll and Violante (2018) focus on the role of the different marginal propensities to consume (MPC) across households by incorporating both liquid and illiquid assets, and Gornemann, Kuester and Nakajima (2016) build an incomplete asset market model where matching frictions create countercyclical labor-market risks.

[5]Kim (2017) finds that an economy when there are more low-income consumers has a smaller impact of monetary policy on real output, but Cravino, Lan and Levchenko (2018) conclude that a change in the level of inequality does not significantly impact the responses of aggregate prices and output to monetary policy. The main result in the paper contradicts that in Kim (2017) or Cravino, Lan and Levchenko (2018): a more equal economy is associated with more effective monetary policy in terms of output. The finding in this study is supported by empirical papers in the literature such as Alpanda and Zubairy (2017) and Voinea, Lovin and Cojocaru (2018).

[6]In this class of the model, (i) money is directly transferred from a government (or a money authority) to households, (ii) money growth (money supply) shocks are introduced as an instrument of the monetary policy, and (iii) households should hold cash to buy goods and services.

a monetary transmission mechanism for both aggregate and disaggregate effects. Due to the nominal wage contracts, real wages will reasonably respond to money supply changes. This will allow firms to change labor demand and affect employment, income, and consumption decisions of households. Therefore, under the nominal wage contract, labor market dynamics is more important in the transmission mechanism of monetary policy actions rather than intertemporal substitution effects. Following Hansen (1985), Rogerson (1988) and Chang and Kim (2007), it is assumed that a household indivisibly decides hours of work.[7] The indivisible labor supply assumption in the heterogeneous-agent model can endogenously generate aggregate labor supply curves. The degree of heterogeneity in the economy matters for the shape of reservation wage distributions, thereby affecting the response of real wages and prices to monetary policy shocks. Thus, aggregate labor supply elasticity, which represents the shape of reservation wage distributions, can be the main linkage between the long-run level of inequality and the effectiveness of monetary policy.

To the best of my knowledge, this model framework is the first that considers a monetary neoclassical model with nominal wage contracts, incomplete asset markets, and rich household heterogeneity.

One of the main findings is that expansionary monetary policy reduces income, consumption, and wealth inequalities. In response to expansionary monetary policy shocks, the nominal wage contract leads to a fall in real wages,[8] and this allows firms to hire more workers. A rise in employment from the bottom of distributions decreases inequality. There are heterogeneous effects of monetary policy on income across the wealth distribution. The different responses of real wages and real asset returns generate compositional effects of mon-

[7]An interplay between a discrete labor choice and wage rigidity will amplify the transmission of monetary policy shocks to the labor market.

[8]There are abundant empirical studies suggesting that real wages are coutercyclical in response to monetary policy shocks. For example, Spencer (1998) provides empirical findings that the real wage response is strongly and robustly negative in response to demand shocks, which supports sticky-wage theories of the business cycle including the model economy in this paper. Similarly, Balmaseda, Dolado and Lopez-Salido (2000) empirically document that real wages are countercyclical in response to aggregate demand shocks. Leiderman (1983) also finds that real wage response to an unanticipated increase in money growth is weakly negative.

etary policy on consumers' income. The rich benefit from a rise in the real asset returns while the poor benefit from an increase in employment in a relative sense. There are also asymmetric responses of consumption between the poor and the rich. Households in the bottom of the wealth distribution increase consumption while asset-rich households tend to decrease consumption. This implies that inflation hurts wealthy households more.[9]

A second important finding is that the long-run level of inequality matters for the effectiveness of monetary policy. The degree of heterogeneity in an economy would determine the size of aggregate labor supply elasticity (or the shape of the reservation wage distribution), which affects the responses of real wages and prices to monetary policy shocks. In an economy with less heterogeneity, labor supply elasticity is larger (or the reservation wage distribution is more concentrated), which implies that relatively more equal households are placed around the market wage. Accordingly, the general equilibrium effect would be bigger: there is a larger decrease in real wages as the price level increases by more, which leads to a bigger response of output. Therefore, a more equal society is associated with more effective monetary policy in terms of output, *ceteris paribus*. I also provide empirical evidence for this model result using state-level panel data: the effects of monetary policy shocks on GDP are larger for low-inequality states.

This paper is organized as follows. Section 2 summarizes the related literature. Section 3 introduces the heterogeneous-agent model economy with indivisible labor and nominal wage contracts. The distributional consequences of monetary policy are discussed in Section 4. Section 5 examines the role of the long-run level of inequality in the effectiveness of monetary policy. Section 6 concludes.

4.2 Related Literature

Some recent papers provide an insightful analysis of redistributive effects of monetary policy. Auclert (2017) investigates redistribution channels of an exogenous monetary policy

[9]This finding is consistent with Doepke and Schneider (2006). Their empirical results suggest that rich and old households hurt more from inflation. Similarly, Wong (2018) finds that consumption of younger people is more responsive to rate shocks.

shock and finds that the effects of monetary policy on aggregate consumption tend to be amplified by such redistribution channels. Kaplan, Moll and Violante (2018) introduce financial market incompleteness in New Keynesian models. In the model of Kaplan, Moll and Violante (2018), there are two types of assets with different degrees of liquidity and different returns. Their model successfully reproduces a wealth distribution, also across liquid and illiquid assets, and a distribution of the marginal propensities to consume (MPC). In this setting, Kaplan, Moll and Violante (2018) find that the indirect effects of an unexpected fall in interest rates, which operate through a general equilibrium (e.g., an increase in labor demand), dominate the direct effects, which are mainly from intertemporal substitution effects. Gornemann, Kuester and Nakajima (2016) consider a model economy in which households differ in their employment status, earnings, and wealth in the context of New Keynesian frameworks and study the importance of the earnings and income composition channels. In their model, matching frictions create countercyclical labor-market risk which is endogenous to monetary policy. Gornemann, Kuester and Nakajima (2016) find that contractionary monetary policy shocks increase inequality since a contractionary shock tends to prolong unemployment spells, as firms reduce labor demand. Another interesting result that they find is that a majority of households prefer substantial stabilization of unemployment even if this means deviations from price stability.[10] The current paper differs from previous studies in the literature on the transmission mechanism of monetary policy in the presence of incomplete markets in that (i) this study focuses more on the distributional effects of discretionary changes in monetary policy actions;[11] (ii) the key monetary transmission mechanism is generated by employment dynamics induced by nominal wage contracts.[12] There are also some empirical analyses that study how the actions of the monetary authority affect inequality,

[10]Werning (2015) also studies some of the channels through which heterogeneity and incomplete markets imply a departure from the aggregate implications of the standard representative agent approach.

[11]The main focus of Gornemann, Kuester and Nakajima (2016) is on the heterogeneous welfare effects of *systematic* monetary policy while Kaplan, Moll and Violante (2018) decompose the transmission mechanism of monetary policy into direct and indirect general equilibrium effects on aggregate consumption.

[12]The countercyclical labor-market risks created by matching frictions are important for the transmission of monetary policy in Gornemann, Kuester and Nakajima (2016) whereas the key features in Kaplan, Moll and Violante (2018) are multiple assets with different degrees of liquidity and different returns.

and contributions are Furceri, Loungani and Zdzienicka (2016) and Coibion et al. (2017).[13] Their main finding is that a contractionary monetary policy shock increases inequality, which is consistent with the model result in this paper.

Most of the previous literature regarding the relation between inequality and monetary policy have mainly focused on one direction of the relation, *the impact of monetary policy on inequality.* The other direction, *the effect of the long-run level of inequality on the effectiveness of monetary policy*, is relatively unstudied. There are a few studies on the role of inequality in the effectiveness of monetary policy. Using retail scanner and consumer panel data, Kim (2017) empirically documents that low-price brands change prices more frequently than high-price brands, and demand for low-price brands has a negative correlation with household income. Using a menu-cost model with vertically differentiated products and heterogeneous consumers, he concludes that the impact of monetary policy on real output decreases when there are more low-income consumers in the economy. Similarly, Cravino, Lan and Levchenko (2018) provide empirical evidence that the estimated impulse responses of high-income households' consumer price indices are lower than those of the middle-income households. Based on a quantitative New-Keynesian model with heterogeneous households where sectors are heterogeneous in their frequency of price changes, they conclude that a realistic change in inequality does not substantially affect the responses of aggregate prices and output to monetary policy. The current paper contributes to the literature by emphasizing the role of different shapes of reservation wage distributions depending on a degree of inequality in the causal relation between inequality and the effectiveness of monetary policy, while Kim (2017) and Cravino, Lan and Levchenko (2018) focus on the channels of different frequency of price changes and heterogeneous composition of consumption goods across households. Because of the different key mechanism from those in Kim (2017) and Cravino, Lan and Levchenko (2018), the main result in the paper contradicts theirs but is supported

[13] Doepke and Schneider (2006) also empirically document the impacts of inflation through changes in the value of nominal assets. Their main finding is that that rich and old households are the main losers from the occurrence of inflation.

by empirical papers in the literature such as Alpanda and Zubairy (2017) and Voinea, Lovin and Cojocaru (2018).[14] This paper is also related to the literature which discusses the relation between the long-run level of inequality and government spending multiplier (Brinca et al., 2016; Yang, 2017)[15] and studies which investigate an importance of micro-level heterogeneity in the propagation of aggregate shocks (e.g., Werning (2015) and Krueger, Mitman and Perri (2016)).[16]

This study also contributes to the literature by incorporating uninsurable risks into a standard real business cycle model with money. The monetary neoclassical models based on the *representative agent* have had a lot of success in accounting for many important macroeconomic issues (e.g. Cooley and Hansen (1989), Cho and Cooley (1995), Janko (2008), and Floden (2000)).[17] However, all these representative-agent models cannot study any macroeconomic questions regarding distributions and inequality. By introducing uninsurable risks and household heterogeneity into a monetary neoclassical model, the current paper investigates not only a propagation of monetary policy shocks to aggregate variables but also the distributional effects of monetary policy actions. Recent studies in the literature, including Ragot (2014) and Camera and Chien (2014), also introduce idiosyncratic shocks in the context of the business cycle models with money. Relative to this literature, the current

[14] Using Romanian data, Voinea, Lovin and Cojocaru (2018) find a contract result that lower inequality is associated with stronger effectiveness and higher homogeneity of monetary policy transmission. Alpanda and Zubairy (2017) also provide empirical evidence that the effects of monetary policy are less powerful during periods of high household debt.

[15] Brinca et al. (2016) study the relation between wealth inequality and the magnitude of fiscal multipliers. Similarly, Yang (2017) investigates the relationship between income inequality and the local government spending multipliers using rich historical state-level data on military procurement and inequality.

[16] Werning (2015) argues that the effect of market incompleteness on the interest rate elasticity of aggregate demand depends on the cyclicality of liquidity and of income risk. Krueger, Mitman and Perri (2016) study the importance of household heterogeneity for aggregate consumption and output dynamics.

[17] Cooley and Hansen (1989) consider cash-in-advance constraints in the standard real business cycle model to analyze if the business cycle depends on inflation states. Cho and Cooley (1995) extend the model of Cooley and Hansen (1989) by introducing nominal wage contracts and study the quantitative importance of the wage contracts for business cycle fluctuations. In the context of Cho and Cooley (1995), Janko (2008) adds labor adjustment costs into a model with nominal wage rigidity and finds that the model with labor adjustment costs is able to replicate reasonable volatilities of real variables and a countercyclical productivity. Floden (2000) constructs a business cycle model economy in which money supply is set to minimize the volatility of inflation and output and finds that small changes in the preferences of a central bank is able to produce large changes in correlations between real and nominal variables.

paper focuses more on the distributional effects of monetary policy.[18]

4.3 Model Economy

I develop a simple dynamic stochastic general equilibrium (DSGE) model which employs a continuum (measure one) of heterogeneous households, a government, and many identical firms. In the model economy, there are four main assumptions. First, asset markets are incomplete as in Huggett (1993) and Aiyagari (1994) in that households cannot fully insure against their idiosyncratic productivity shocks. This assumption along with borrowing constrains helps generate substantial heterogeneity across characteristics of individual households including wealth, income, employment, and consumption. Second, following Hansen (1985), Rogerson (1988) and Chang and Kim (2007), it is assumed that a household indivisibly decides hours of work. As is well-known, extensive margins for time devoted to work play an important role in accounting for the variation in total hours worked. Particularly, once indivisible labor is employed in the incomplete asset market model, the aggregate labor supply elasticity depends on how different individual reservation wages are compared to the market wage (Chang and Kim, 2006). Third, cash-in-advance constraints are considered in the standard real business cycle model as in Cooley and Hansen (1989). In this setup, households should hold money to purchase goods and services. Lastly, as in Cho and Cooley (1995) and Janko (2008), nominal wage contracts are incorporated into the model economy to produce a reasonable propagation of monetary policy shocks. Under the nominal wage contract, labor market dynamics is relatively important in the transmission mechanism of monetary policy actions rather than intertemporal substitution effects.

[18] Sterk and Tenreyro (2018) also investigate a redistribution channel for the transmission of monetary policy focusing on durable purchases using the heterogeneous-agent model with money, but their model is based on flexible prices.

4.3.1 Environment

4.3.1.1 Households

Each household maximizes her expected lifetime utility over cash goods c_{1t}, credit goods c_{2t}, hours of work h_t, shown as:

$$\max \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\psi \log c_{1t} + (1-\psi) \log c_{2t} - \chi \frac{h_t^{1+1/\phi}}{1+1/\phi} \right) \right]$$

subject to

$$P_t c_{1t} \leq m_{t-1} + tr_t, \tag{4.1}$$

$$P_t(c_{1t} + c_{2t} + a_{t+1}) + m_t = W_t^c x_t h_t + (P_t + R_t) a_t + m_{t-1} + tr_t, \tag{4.2}$$

and

$$a_{t+1} \geq \bar{b},$$

where $0 < \beta < 1$ denotes the time discount factor, $\chi > 0$ is a parameter for disutility from working, ψ is a weight for cash goods, and ϕ represents labor supply elasticity. There are two types of consumption goods in the model economy: one is a cash good (denoted by c_{1t}), and the other is a credit good (denoted by c_{2t}). When a household wants to purchase cash goods, she is required to hold money. The currency holdings of a household consist of two sources: money balances carried over from the previous period, m_{t-1}, and a money transfer from the government, tr_t. Specifically, a decision on cash goods for a households must satisfy cash-in-advance (CIA) constraints as in Equation 4.1, where P_t is the price level. As in Cooley and Hansen (1989) and others in the literature, it is assumed that the CIA constraint holds with equality.[19]

In addition to the CIA constraint, each household faces the budget constraints as in Equation 4.2. It is assumed that a household is endowed with one unit of physical time, which she allocates between hours worked and leisure. When a household works for h_t amount of hours, she earns $W_t^c x_t h_t$ as nominal wage earnings, where W_t^c is the nominal contract wage rate for the efficiency unit of labor, and x_t denotes her labor productivity. Importantly, it is assumed that a labor supply decision made by a household is indivisible following Hansen (1985), Rogerson (1988), and Chang and Kim (2007): a household supplies a fixed amount of hours ($h_t = \bar{h}$), or she does not work at all ($h_t = 0$). Accordingly, there are two employment status for each household: employment and non-employment. A household can save or borrow by trading a claim for financial assets, a_t, which yields the nominal rate of return, R_t. The asset markets are incomplete following Huggett (1993) and Aiyagari (1994): households cannot issue any assets contingent on their future idiosyncratic risks x. A household faces a borrowing constraint that limits the fixed amount of debt: the assets holding, a_{t+1}, cannot go below $\bar{b}$ at any time. Since the CIA constraint binds all the time, and cash goods (c_{1t}) are determined by the CIA constraint, a household makes decisions of credit goods (c_{2t}), asset holdings (a_{t+1}), and money balances (m_t), given the budget constraint. It is assumed that labor productivity, x, follows a stochastic process with transition probabilities $P_x(x'|x) = Pr(x_{t+1} = x'|x_t = x)$ and an AR(1) process in logs:

$$\ln x' = \rho_x \ln x + \varepsilon_x, \quad \varepsilon_x \sim N(0, {\sigma_x}^2).$$

Since households face borrowing constraints and asset-market incompleteness, they will behave differently in response to monetary policy shocks, depending on individual state variables.

[19] As discussed by Cooley and Hansen (1989), a sufficient condition for the CIA to be always binding is that the time discount factor is smaller than the gross growth rate of money supply. In the standard calibration case, this condition is generally satisfied.

4.3.1.2 Firms

The production technology for the representative firms is represented by the function given by:

$$F(K, L, Z) \equiv ZK^{\alpha}L^{1-\alpha},$$

where Z, K, L and α denote aggregate productivity, aggregate capital, aggregate effective labor, capital income share, respectively. Given the contract wages, the representative firm makes decisions for labor and capital demand to maximize current profits such that:

$$\Pi_t = \max_{K_t,L_t} \left\{P_t Z_t K_t^{\alpha} L_t^{1-\alpha} - W_t^c L_t - R_t K_t - P_t \delta K_t)\right\},$$

where δ is the real depreciation rate for capital.

4.3.1.3 The Government

The government takes up the role of printing money and injecting it into the economy, and it faces the constraint:

$$P_t G_t + T_t = M_t - M_{t-1},$$

where G_t is real government expenditure, T_t is total nominal transfer, and M_t is money supply. Since this study does not focus on the impact of changes in government spending, it is assumed that G_t is constant over time.[20] Accordingly, the total nominal transfer can be directly financed by changing money supply. The government issues money according to the following rule:

$$M_{t+1} = g_{t+1} M_t,$$

where g_{t+1} is the gross growth rate of money between periods t and $t+1$. g_{t+1} is assumed to evolve according to the autoregression of the form:

$$\log(g_{t+1}) = \rho_g \log(g_t) + \omega_{t+1},$$

[20] Without loss of generality, G_t is assumed to be zero in every time period.

and

$$\omega_{t+1} \sim N((1-\rho_g)\log(\bar{g}), {\sigma_g}^2),$$

where $\bar{g}$ represents the gross growth rate of money in the steady state. As mentioned above, $\mathbb{E}_t[1/g_{t+1}] < 1/\beta$ implies that the CIA constraint is binding in every period.

4.3.1.4 Nominal Wage Contract

A nominal rigidity is introduced into the model economy in order to obtain reasonable effects of monetary policy and to see the importance of labor market for the propagation of monetary policy. In the model economy, nominal wage rigidity arises from nominal wage contracts agreed to by households and firms. In this paper, a "contract" means that households and firms set up rules and agree to follow them. There are three rules in the wage contract: a wage setting rule, a labor input setting rule, and an employment allocation rule. The first two are agreements between the firms and households while the last one is a rule among households. The wage setting rule and the labor input setting rule will be discussed in this section, and the employment allocation rule will be followed the households' problem.

4.3.1.4.1 Wage Setting Rule The nominal contract wage rate is determined by both desired wages and previous contract wages. I use a natural formulation of the contract wages as a weighted average (in logs) of the wage in the previous period and the desired wage. Formally, the nominal contract wage rate, W_t^c, is given by:

$$W_t^c = (W_t^o)^{1-\lambda}\left(W_{t-1}^c\right)^{\lambda}, \tag{4.3}$$

where W_t^o is the nominal desired wage rate, and λ is the nominal wage rigidity index. Under this assumption, an important issue is how to determine the desired nominal wages. Regarding this issue, I assume that the nominal desired wages follow the pricing rule from the frictionless economy where nominal wage contracts are omitted (Cho and Cooley, 1995;

Janko, 2007). Specifically, as in Chang and Kim (2007), the forecasting function for the market wage rate is assumed to take log-linear functions of aggregate state variables, K_t and g_t.[21] Hence, the nominal desired wage, W_t^o, is given by:

$$\log(W_t^o) = \gamma_1 + \gamma_2 \log(K_t) + \gamma_3 \log(g_t), \tag{4.4}$$

where K_t is the aggregate capital. Note that the coefficients (γ_1, γ_2, and γ_3) in the forecasting function are estimated from the economy without the nominal wage contract. The assumption of desired wages has two interesting features. First, even if the coefficients are exogenously given, the path of nominal desired wages is endogenously determined since the series of aggregate capital, K_t, is an outcome in the economy with nominal wage contracts. Second, if there is no a wage contract in the model economy ($\lambda = 0$), then the desired wage collapses to the market-equilibrium wage in the frictionless economy, and so does the contract wage.

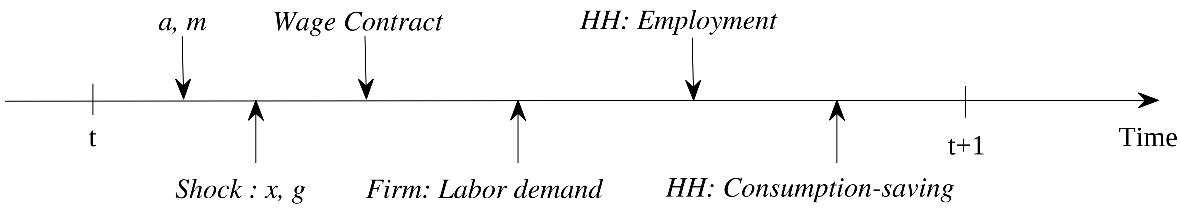
Labor Input Setting Rule Since the contract wage rates are not equal to the equilibrium ones, it is also important who decides the aggregate labor input. Following Cho and Cooley (1995), Floden (2000), and Janko (2008), I assume that it is the representative firm that determines aggregate effective labor once aggregate shocks are realized. In other words, by agreeing to a nominal contract wage, households cede the right to choose the total efficiency unit of labor to the firms.

4.3.1.5 Timing

The timing assumptions of the model is summarized in Figure 4.1. Households enter with their own asset and money holdings (a, m) brought from the previous period. Then individual labor productivity shocks, x, and the aggregate money supply shocks, g, are

[21] As found in Krusell and Smith (1998), a very high precision can be obtained by approximating the distribution across characteristics of households using the first moment (mean asset, K) of it.

Figure 4.1: Timing of Event



Note: The figure presents the timing of the event for the model economy. "HH" denotes "households."

realized. Observing these shocks, firms and households conclude a nominal wage contract. Given the nominal contract wage, W^c, the firms make decisions on aggregate effective labor, L^c. Households then make decisions of their employment first and determine consumption, asset and money holdings next. The details on the households' problem will be discussed in the next subsection.

4.3.2 Household Problem

Under the wage contract, households should provide the efficiency units of labor that the firm demands since they agree to cede the right to decide the aggregate effective labor to the representative firm. In the representative-agent model, it is easy to fulfill the wage contract for the representative household since all identical households are employed and supply the same amount of labor. In the heterogeneous-agent model economy, however, the "employment allocation rule"—how different households allocate their hours—matters given the aggregate effective labor and the contract wages since reservation wages are different across households. Since the employment status of a household affects her consumption and saving decisions, a reasonable assumption of labor supply decisions for households is a critical issue. To this end, I introduce two sequential steps for households' problems. The first step is the employment allocation rule. I assume that in the first step, given the wage contract (or the aggregate effective labor), households make a decision on their employment. In the second step, given the wage contract and the employment decisions, households decide how much they consume and save. The two steps are summarized as follows:

- **Step 1** (**Employment decision**): given the wage contract, households make a decision on employment (the employment allocation rule).

- **Step 2 (Consumption-saving decision)**: given the wage contract and employment decisions, households make consumption-saving decisions.

4.3.2.1 Reservation Wages

Before discussing how households allocates hours in the model economy with the nominal wage contract, it is useful to consider households' employment decisions in the frictionless economy. In the model economy with indivisible labor, a reservation wage rate (per effect labor) is an important concept for households to make their employment decisions. Define θ and Θ as the vector of individual and aggregate state variables, respectively: $\theta \equiv (a, m, x)$ and $\Theta \equiv (\mu, g)$, where $\mu(\theta)$ is the type distribution of households.[22] Let me consider the labor supply decision for a household. Since the labor supply choice is assumed to be indivisible ($h_t = \overline{h}$ or $h_t = 0$), there are two employment status for a household: a household is employed or non-employed. The value function for an employed household, denoted by $V^E(\theta, \Theta)$, is defined as:

$$V^E(\theta, \Theta) = \max_{c_1, c_2, a', m'} \left\{ \psi \ln c_1 + (1-\psi) \ln c_2 - \chi \frac{\overline{h}^{1+1/\phi}}{1+1/\phi} + \beta \mathbb{E} \left[V(\theta', \Theta') \right] \right\}$$

subject to

$$Pc_1 \leq m + tr,$$

$$P(c_1 + c_2 + a') + m' = W^M x \bar{h} + (P+R)a + m + tr, \; a' \geq \bar{b},$$

and

$$\mu' = \Psi(\Theta),$$

[22]Denote $\mathcal{A}$, $\mathcal{M}$, and $\mathcal{X}$ for sets of all possible realizations of a, m and x, respectively. Then, the measure $\mu(a, m, x)$ is defined over a σ-algebra of $\mathcal{A} \times \mathcal{M} \times \mathcal{X}$.

where W^M is the nominal market wage rate, and Ψ denotes a forecasting function for μ. To simplify notation, time subindices are suppressed, and primes denote variables in the next period. The value function for a non-employed household, denoted by $V^N(\theta, \Theta)$, is defined as:

$$V^N(\theta, \Theta) = \max_{c_1, c_2, a', m'} \{\psi \ln c_1 + (1-\psi) \ln c_2 + \beta \mathbb{E}[V(\theta', \Theta')]\}$$

subject to

$$Pc_1 \leq m + tr,$$

$$P(c_1 + c_2 + a') + m' = (P + R)a + m + tr,\ a' \geq \bar{b},$$

and

$$\mu' = \Psi(\Theta).$$

Then, the employment decision, $h(\theta, \Theta)$, for a household is:

$$\max_{h \in \{0, \bar{h}\}} \left\{V^E(\theta, \Theta), V^N(\theta, \Theta)\right\}. \tag{4.5}$$

The reservation wage rate, denoted by $W^R(\theta, \Theta)$, is an individual subjective nominal wage rate (per effective labor) which makes values when working and not working indifferent. Formally, under $W^R(\theta, \Theta)$, a value function for an employed household, $V^E(\theta, \Theta)$, and a value function for a non-employed household, $V^N(\theta, \Theta)$, are the same. In the frictionless economy, a household is employed if the nominal market wage rate (per effective labor), W^M, is equal to or larger than her nominal reservation wage rate (per effect labor), $W^R(\theta, \Theta)$. That is,

$$h(\theta, \Theta) = \begin{cases} \bar{h} & \text{if } W^M \geq W^R(\theta, \Theta) \\ 0 & \text{otherwise} \end{cases}. \tag{4.6}$$

Importantly, the reservation wage rate is different across households, depending on net wealth and productivity.[23] Intuitively, $W^R(\theta, \Theta)$ is a decreasing function of labor productivity, x, and an increasing function of asset holdings, a.[24] Accordingly, a household with a lower reservation wage, or with higher productivity and smaller amount of assets is more likely to be employed in the frictionless economy.

4.3.2.2 Employment Decision (Employment Allocation Rule)

Next, the employment allocation rule is discussed as follows: how heterogeneous households determine their employment under the wage contract. Since the contract wage depends on the contract wage in the previous period, now define $\Theta \equiv (\mu, g, W^C_{-1})$, where W^C_{-1} is the contract wage in the previous period. I assume that given the wage contract, households make a decision on their employment first before making consumption-saving decisions. By agreeing to cede the right to decide an aggregate efficient labor to the representative firm, households should provide effective labor that the firm demands. The aggregate effective labor associated with W^c (the current nominal contract wage) is denoted by L^c. Suppose that $\widetilde{W}$ is the wage rate at which households endogenously provide effective labor of L^c. Note that $\widetilde{W}$ is different from W^c but a function of W^c. In order to fulfill the wage contract, a household, whose reservation wage rate is less than or equal to $\widetilde{W}$, should work. Formally, now the employment decision for a household is given by:

$$h(\theta, \Theta) = \begin{cases} \bar{h} & \text{if } \widetilde{W} \geq W^R(\theta, \Theta) \\ 0 & \text{otherwise} \end{cases}. \tag{4.7}$$

In other words, households solve Equation 4.5 under $\widetilde{W}$ instead of W^M.[25] With the fulfillment of the wage contract, the following condition must be satisfied:

[23]Of course, it is also dependent on aggregate states in the economy.

[24]In the next section, I will discuss the distribution of the reservation wage rate over individual state variables.

[25]Note that $V(\theta, \Theta)$ is determined in the second step (after $h(\theta, \Theta)$ is obtained) in the model with the wage contract while $V(\theta, \Theta)$ and $h(\theta, \Theta)$ will be jointly determined in the frictionless economy.

$$L^c(\Theta) = \int xh(\theta, \Theta)d\mu.$$

It should be noted that even if employment decisions are made endogenously under $\widetilde{W}$, some are voluntarily (non-)employed while others are involuntarily (non-)employed because of the wage contracts. Of course, the size of the involuntary labor choices depends on aggregate states in the economy and the wage rigidity index. As discuss above, this is a reasonable assumption of employment decisions for households is that households are employed in ascending order by their reservation wage rates (per effective labor) until they provide L^c as in the economy with no wage contracts.

4.3.2.3 Consumption-saving Decision

In the second step, given the wage contract and the employment decisions, households decide how much they consume and save. It is assumed that given employment status, $h(\theta, \Theta)$, decisions for c_1, c_2, a', and m' are made under the contract wage, W^c, such that:

$$V(\theta, \Theta) = \max_{c_1, c_2, a', m'} \left\{ \psi \ln c_1 + (1-\psi)\ln c_2 - \chi \tfrac{h(\theta,\Theta)^{1+1/\phi}}{1+1/\phi} + \beta \mathbb{E}\left[V(\theta', \Theta')\right] \right\}$$

subject to

$$Pc_1 \leq m + tr,$$

$$P(c_1 + c_2 + a') + m' = W^c xh(\theta, \Theta) + (P + R)a + m + tr, \; a' \geq \bar{b},$$

and

$$\mu' = \Psi(\Theta).$$

To sum up, the employment decisions, $h(\theta, \Theta)$, are made under $\widetilde{W}$ to fulfill the wage contract (to supply the effective labor that the firm demands), while consumption-investment decisions $(c_2(\theta, \Theta), a'(\theta, \Theta)$ and $m'(\theta, \Theta))$ are made under W^c.

4.3.3 Definition of Equilibrium

A recursive competitive equilibrium is a transition operator $\Psi(\Theta)$, a set of factors $\{K(\Theta), L^c(\Theta)\}$, a set of value functions $\{V^E(\theta,\Theta)$,$V^N(\theta,\Theta)$, $V(\theta,\Theta)\}$, a set of prices $\left\{W^C(\Theta), \widetilde{W}(\Theta), P(\Theta), R(\Theta)\right\}$, and a set of policy functions $\{c_1(\theta,\Theta), c_2(\theta,\Theta)$,$m'(\theta,\Theta)$,$a'(\theta,\Theta)$,$h(\theta,\Theta)\}$ such that:

1. Nominal wage contracts

   - Contract wage setting rule:

   $$W^c = (W^o)^{1-\lambda}\left(W^c_{-1}\right)^{\lambda},$$

   - A household, whose reservation wage rate is less than or equal to $\widetilde{W}$, should work, i.e.,

   $$h(\theta,\Theta) = \bar{h} \ \text{ if } \widetilde{W} \geq W^R(\theta,\Theta).$$

   - Households supply the effective labor that the firm demands:

   $$L^c(\Theta) = \int xh(\theta,\Theta)d\mu.$$

2. Individual optimization: given W^c and $h(\theta,\Theta)$, optimal decision rules $c_1(\theta,\Theta)$, $c_2(\theta,\Theta)$, $a'(\theta,\Theta)$, and $m'(\theta,\Theta)$ solve the Bellman equation, $V(\theta,\Theta)$.

3. The firm's profit maximization: given W^c, $K(\Theta)$ and $L^c(\Theta)$ satisfy $F_L(K,L^c) = \frac{W^C(\Theta)}{P(\Theta)}$ and $F_K(K,L^c) - \delta = \frac{R(\Theta)}{P(\Theta)}$ for all Θ.

4. Markets clearing: for all Θ,

   - capital market clearing: $K(\Theta) = \int ad\mu$, and
   - goods market clearing: $ZK(\Theta)^{\alpha}L^c(\Theta)^{1-\alpha} = C(\Theta)+I(\Theta)$ where $C(\Theta) = \int c_1(\theta,\Theta)d\mu + \int c_2(\theta,\Theta)d\mu$, and $I(\Theta) = K'(\Theta) - (1-\delta)K(\Theta)$.

5. Balanced budget of the government: $PG + T = M' - M$.

6. Consistency of individual and aggregate behaviors: for all $A^0 \subset \mathcal{A}$, $M^0 \subset \mathcal{M}$, and $X^0 \subset \mathcal{X}$,

$$\mu'(A^0, M^0, X^0) = \int_{A^0,M^0,X^0} \left\{ \int_{\mathcal{A},\mathcal{M},\mathcal{X}} \mathbf{1}_{a'=a'(\theta,\Theta),m'=m'(\theta,\Theta)} dP_x(x'|x) d\mu \right\} da'dm'dx'.$$

4.3.4 Calibration

In this section, I discuss calibration for the parameters used in the model economy. A simulation period is a quarter in the model. Table 4.1 summarizes the parameter values used in the model economy.

4.3.4.1 Preference

The parameter ϕ, which represents the micro elasticity of labor supply, is set to 0.4. This value is based on the findings that conventional micro estimates of the elasticity of labor supply are small $(0 - 0.5)$. Since labor supply is assumed to be discrete, the value of ϕ does not affect the aggregate labor-supply elasticity, and it is determined by the shape of the reservation-wage distribution. According to the Michigan Time-Use Survey, a typical household spends around one third of her discretionary time for working. Hence, fixed amount of hours worked, $\overline{h}$, is chosen to be 1/3. The time discount factor, β, and the disutility parameter of working, χ, are set so that quarterly return to capital is one percent, and the employment rate is 60 percent, respectively.[26] The U.S. data such as the PSID and Survey of Consumer Finances (SCF) consistently report that employment rates are around 60 percent.[27] According to Bagnall et al. (2016), the share of payments by value done with cash by US consumers is 23 percent, and the share of average transaction values done with cash is 17.8 percent. Based on these measures, the parameter for the share of cash goods, ψ, is chosen to be 20 percent.

[26]The discount factor, β, in the heterogeneous-agent model is smaller than that in the representative-agent model, because market incompleteness makes households increase precautionary savings as noted in Aiyagari (1994).

[27]Self-employed workers are excluded for the calculation of employment rates.

Table 4.1: Parameters of the Model Economy

Parameter	Value	Description	Source/Target Moments
		Literature or Data Directly	
$\overline{h}$	1/3	Extensive margin for hours worked	Standard
ϕ	0.4	Labor supply elasticity	Standard
ψ	0.20	Share of cash goods	Bagnall et al. (2016)
ρ_x	0.939	Persistence of x shocks	Chang, Kim and Schorfheide (2013)
σ_x	0.287	Standard deviation of x shocks	Chang, Kim and Schorfheide (2013)
δ	0.025	Capital depreciation rate	Standard
α	0.36	Capital income share	Standard
ρ_g	0.8	Persistence of g shocks	Christiano (1991)
σ_g	0.004	Standard deviation of g shocks	Christiano (1991)
$\bar{g}$	1.012	Steady state g	Janko (2008)
λ	0.8	Wage rigidity index	Smets and Wouters (2003)
		Calibrated	
β	0.97782	Time discount factor	Real return to capital
$\overline{b}$	-3.0	Borrowing constraint	Doubled average quarterly earnings
χ	145.8	Parameter for disutility from working	Employment rate

Note: The table summarizes the parameter values used in the model economy. Parameters in the upper panel are from the literature or data directly while parameters in the bottom panel are chosen to match data moments.

4.3.4.2 Production and Borrowing Constraint

Parameter values for production are quite standard. Since this study does not focus on the impact of aggregate productivity shocks, it is assumed that Z_t is constant at 1 over time. The capital income share, α, and the quarterly depreciation rate, δ, are calibrated to be 0.36 and 2.5 percent, respectively. The borrowing constraint, $\overline{b}$, is -3.0, which is approximately twice of the quarterly average earnings in the model economy.[28] With this value, the fraction of households who own zero or negative wealth is around 20 percent, which is consistent with that in the U.S. data such as the PSID.

[28]Similarly, Chang and Kim (2006) also choose the borrowing constraint to target one and a half of quarterly earnings for a household who has the average-productivity.

Table 4.2: Transition Probabilities for Money Supply Shock

		t+1				
		g_1	g_2	g_3	g_4	g_5
	g_1	0.40	0.37	0.18	0.05	0.01
	g_2	0.22	0.35	0.29	0.12	0.03
t	g_3	0.09	0.24	0.34	0.24	0.09
	g_4	0.03	0.12	0.29	0.35	0.22
	g_5	0.01	0.05	0.18	0.37	0.40

Note: Transition probabilities per quarter. g_i denotes a money growth grid i. Rounding for the table means rows may not sum to 1.

4.3.4.3 Labor Productivity

For individual labor productivity shocks, previous studies in the literature including Floden and Linde (2001), Chang and Kim (2006), and Chang, Kim and Schorfheide (2013) consistently report that the shocks are persistent, and the variance is also large. Following Chang, Kim and Schorfheide (2013), I set $\rho_x = 0.939$ and $\sigma_x = 0.287$, which are estimated with the AR(1) wage process from the PSID.[29] The number of grids for labor productivity is 15. Grid vectors of productivity are equally spaced in which $\ln x$ lies between $-3\sigma_x\sqrt{1-\rho_x^2}$ and $3\sigma_x\sqrt{1-\rho_x^2}$. Transition probabilities of productivity are approximated using Tauchen (1986) algorithm and are shown in Table **??**.

4.3.4.4 Money Supply and Wage Contract

Regarding the money supply shocks, Christiano (1991) estimate values for the parameters of the money growth process, ρ_g and σ_g, using the U.S. time series data. Following his estimate, I choose $\rho_g = 0.8$ and $\sigma_g = 0.004$. The steady-state rate of gross money growth is assumed to be 1.015.[30] The number of grids for money growth is 5. Grid vectors of productivity are equally spaced in which $\ln g$ lies between $-3\sigma_g\sqrt{1-\rho_g^2}$ and $3\sigma_g\sqrt{1-\rho_g^2}$. Transition probabilities of money growth are also approximated using Tauchen (1986) algorithm and are shown

[29] Chang, Kim and Schorfheide (2013) use the maximum-likelihood estimation (MLE) when they estimate the AR(1) process of wage rates in logs to solve the self-selection problem for wage workers.

[30] This value is very standard in the literature. For example, Janko (2008) find that the estimate of the parameter value is 1.012.

Table 4.3: Key Aggregate Moments

Moment	Data	Model
Targeted Moment		
Employment rate	0.60	0.60
Real capital return	0.01	0.01
Untargeted Moment		
Income Gini	0.57	0.59
Wealth Gini	0.78	0.68
Consumption Gini	0.30	0.33
Flow out of employment	6.82	5.60
Flow into employment	7.01	5.60

Note: Information for income and wealth in the data are from Diaz-Gimenez, Quadrini and Rios-Rull (1997), while statistics for consumption is the Consumer Expenditures Survey (CEX) average for the period 1980-2008. The measure for the employment flows is from the working paper of Chang and Kim (2006).

in Table 4.2. The index for nominal wage rigidity is set to 0.8, which is in the range of the values typically used in the literature. For example, Smets and Wouters (2003) estimate a degree of nominal wage sickness and find that it is 0.74, which is the median in the posterity distribution.

4.4 The Effects of Monetary Policy on Inequality

4.4.1 Aggregate Moments and Cross-sectional Distributions

First, I present the main results of aggregate moments. I summarize the data and the model counterparts of the targeted (upper panel) and untargeted moments (bottom panel) in Table 4.3. The model economy fits well the targeted moments by construction, thus we mainly examine the fit of the model to the data in terms of the untargeted moments. Table 4.4 summarizes the shares of income, wealth and consumption for the data and the model economy. The model economy reasonably replicates the income distribution of the U.S., making the income Gini index of the simulated model 0.59, which is comparable to the data (0.57). The inequality of wealth distribution in the model economy is less concentrated compared to the U.S. data: the model economy makes the wealth Gini index about 0.68

Table 4.4: Three Key Distributions

	Quintile					
	1st	2nd	3rd	4th	5th	Gini
U.S. Data						
Share of Income	2.18	6.63	11.80	19.47	59.91	0.57
Share of Wealth	-0.39	1.74	5.72	13.43	79.49	0.78
Share of Consumption	11.02	15.07	18.07	22.53	33.31	0.30
Model Economy						
Share of Income	0.81	4.57	11.15	23.10	60.37	0.59
Share of Wealth	-2.70	2.00	10.00	24.55	66.15	0.68
Share of Consumption	6.49	12.50	17.57	24.02	39.42	0.33

Note: Information for income and wealth in the data are from the Survey of Consumer Finances (SCF) 1992 in Diaz-Gimenez, Quadrini and Rios-Rull (1997), while statistics for consumption is the Consumer Expenditures Survey (CEX) average for the period 1968-2015.

while it is 0.78 in the U.S. data. Consumption inequality is also well replicated by the model: Gini index for consumption is 0.33 in the model, which is similar to what is observed in the U.S. data (0.30). In the U.S data, on average, 6.82 percent of the population moved from employment to nonemployment each quarter; 7.01 percent of the population moved in the opposite direction, from nonemployment to employment. In the model, these flows are 5.60 percent. Although I do not calibrate the model to match these values, the worker flows are fairly close to those in the U.S. data.

4.4.2 The Aggregate Effects of Monetary Policy

In this subsection, I discuss the transmission of an expansionary one-standard-deviation monetary policy shock (positive money growth shocks). The responses of key aggregate variables in the model economy to a one-standard-deviation monetary policy (money growth) shocks for 20 quarters of horizon are shown in Figure 4.2. The mechanism of the effects of an unexpected rise in money supply on real economic activity in the model is through the nominal wage contract. An unexpected rise in the money growth leads to a higher price

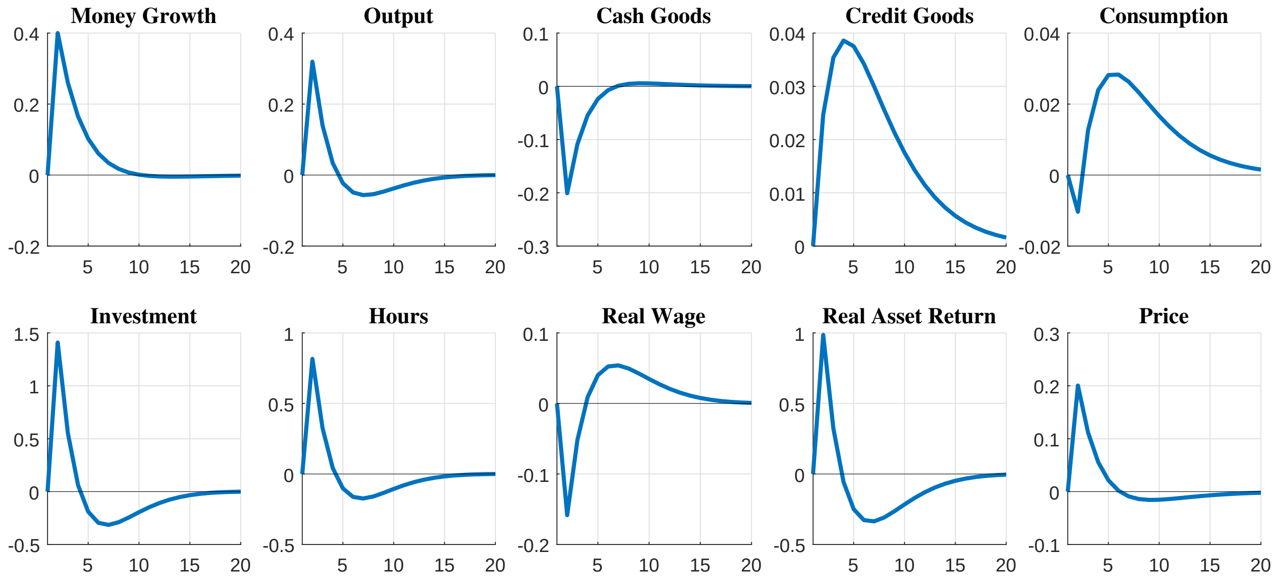


Figure 4.2: Impulse-responses of Aggregate Variables

Note: Impulse response to a one-standard-deviation monetary policy (money growth) shock. All variables other than interest rates are logged. The x-axis shows time since the shock in quarters.

level: it increases by 0.20 percentage.[31] Accordingly, real wages drop (by 0.16 percent), and real capital returns increase (by 0.99 percent points) due to the nominal wage contract. A fall in the rental price for labor allows the firms to hire more workers. Accordingly, hours (employment) and output increase significantly (by 0.82 percent and 0.32 percent, respectively). A rise in price level decreases demand for cash goods, while credit goods increase because of a rise in income, which leads to a smoothly rise in total consumption. Total consumption rises by 0.03 percent at the peak. It should be also noted that a rise in involuntary employment also contributes to an increase in credit goods since households who are forced to work by the wage contract increase their consumption to compensate for increased disutility from working. Investment also increases markedly (by 1.41 percent) due to a portfolio change caused by the higher price level and a rise in the real asset return.[32] The dynamics of most of the macro variables in the model economy is well supported by empirical studies in the literature including Christiano, Eichenbaum and Evans (2005).

[31]Following studies in the literature (e.g., Cho and Cooley (1995) and Floden (2000)), the price level is divided by aggregate money stock to make it stationary.

[32]Of course, a rise in total income also contributes to an increase in investment.

4.4.3 Heterogeneous-agent vs. Representative-agent Models

At this stage in the analysis, one question may arise: what is the difference between the heterogeneous-agent model (HA model) and the representative-agent model (RA model) which has been studied in the previous literature? The basic assumptions for the nominal wage contacts in the RA model are the same as those in the HA model other than the employment decisions for households. In the RA model, households are not able to decide who works or not as in the HA model since they are assumed to be identical. Hence, I assume that in the RA model, given the contract wage, all the identical households provide L^c, which is determined by firm's profit maximization condition. This assumption is widely used in the literature such as Cho and Cooley (1995), Floden (2000), and Janko (2008). For calibration strategy for the RA model, I choose $\beta = 0.99$ to match a one percent quarterly return to capital, and disutility parameter of working, χ, is chosen to target the aggregate hours of 0.2, as in the HA model (60 employment rate$\times 1/3$). Importantly, the labor supply elasticity, ϕ, is chosen to be 1.14, which is the same as the aggregate labor supply elasticity generated by the HA model.[33]

Figure 4.3 presents the impact of a one-standard-deviation expansionary monetary shock for the HA model and the RA model. The solid lines show the response in the economy with heterogeneous households, while the dotted lines report the response in the economy with a representative household. A striking feature in the HA model is that hours increase significantly by more than those in the RA model, even though the model economies are assumed to have the same aggregate labor supply elasticity. The difference is mainly due to how households allocate their hours in the two economies. In both models, by agreeing to a nominal contract wage, households should supply the total efficiency unit of labor that the firms want. What is different in the two models is how households make decisions of their labor supply. In the HA model, it is assumed that households are employed in ascending order by their reservation wage rates (per effective labor) until they provide L^c. Hence,

[33] Chang and Kim (2007) also use the same calibration strategy for their RA model.

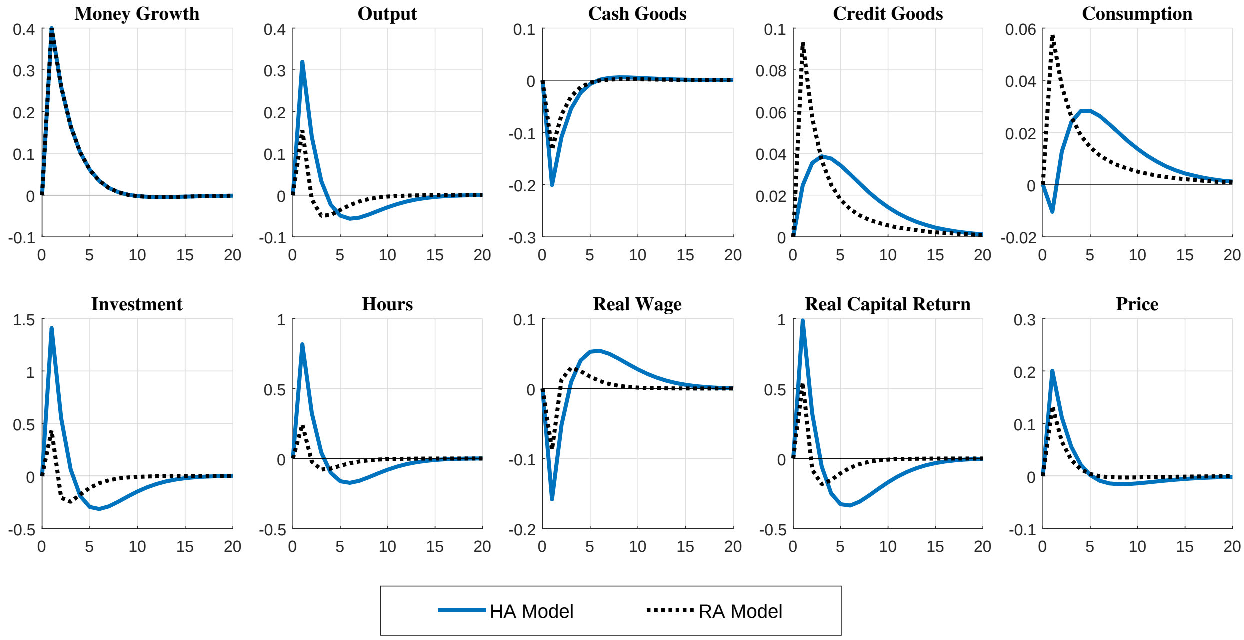


Figure 4.3: Impulse-responses of Aggregate Variables: HA vs. RA models

Note: Impulse response to a one-standard-deviation monetary policy (money growth) shock. All variables other than interest rates are logged. The x-axis shows time since the shock in quarters. The solid blue lines show the response in the economy with heterogeneous households, while the dotted lines report the response in the economy with a representative household.

households whose reservation wage rates are slightly higher than the contract wage tend to be newly employed. Since the reservation wage is a decreasing function of productivity, less productive households newly work. Accordingly, given the demand for aggregate effective labor, more employed households are needed in the HA model to fulfill the wage contract since newly employed households are less productive. In the RA counterpart, however, all the households have the same productivity, which is the average productivity in the HA model. The efficient unit of labor provided by the newly employed households in the RA model is larger than that in the HA model. Hence, hours increase significantly by less in the RA model compared to those in the HA model.

Another interesting feature is that households in the HA model have a smoother consumption path than those in the RA counterpart. Households who are forced to work by the wage contract are likely to increase their consumption to compensate for the reduction of utility from involuntarily working. In the RA model, consumption will increase by more than that in the HA model since every household is forced to work more. However, credits goods do not jump substantially in the HA model since consumption dynamics is not only affected by households who work involuntarily, and precautionary saving behaviors of voluntary workers stop credits goods from increasing by more.[34] As found in Figure 4.3, credit goods in the RA model dramatically increase on impact while those in the HA model have a relatively smooth path. In the RA model, at the cost of a rise in consumption on impact, investment increases by less than that of the HA counterpart. On net, output on impact rises by more in the HA economy than that in the RA economy. Additional general equilibrium effects also play important a role in generating the larger of output in the HA model. A money demand also increases by more in the HA model than that in the RA counterpart due to the smooth consumption response, which leads to a larger increase in the price level and output in the HA model.

4.4.4 Behaviors of Marginal Workers

In response to an unexpected monetary expansion, by agreeing to the nominal wage contract, households whose reservation wage rates are closed to contract wage are employed since they should supply the total efficiency unit of labor that the firms demand. Households who are newly employed because of the wage contract are likely to behave differently from the ones who are voluntarily working. As briefly mentioned above, for example, a rise in involuntary employment contributes to an increase in credit goods since households who are forced to work by the wage contract may increase their consumption to compensate for increased welfare costs from voluntarily working. In this subsection, I discuss the behaviors of this type of households. To this end, let me first define a *marginal worker* in the economy. Define $V^E(\theta, \Theta, \widetilde{W})$ a value function for a employed household under $\widetilde{W}$. $V^N(\theta, \Theta, \widetilde{W})$, $V^E(\theta, \Theta, W^c)$, and, $V^N(\theta, \Theta, W^c)$ can be defined in a similar way. A household is defined as a *marginal worker* if

[34]Intuitively, aggregate response of credit goods depend on the marginal worker distribution in the economy. I will discuss this issue later.

$$V^E(\theta, \Theta, \widetilde{W}) \geq V^N(\theta, \Theta, \widetilde{W}) \text{ } and \text{ } V^E(\theta, \Theta, W^c) < V^N(\theta, \Theta, W^c) \tag{4.8}$$

or

$$V^E(\theta, \Theta, \widetilde{W}) < V^N(\theta, \Theta, \widetilde{W}) \text{ } and \text{ } V^E(\theta, \Theta, W^c) \geq V^N(\theta, \Theta, W^c). \tag{4.9}$$

Equation 4.8 or 4.9 implies that a marginal work does not want to work under the contract wages but should work by the wage contract, or she wants to work under the contract wages but cannot work by the contract. Figure 4.4 reports the responses of key per capita variables for marginal workers and non-marginal workers in response to an unexpected increase in money supply.[35] As expected, the last column of the figure shows that the population of marginal workers increases due to the wage contract. This implies that the aggregate hours variation is largely driven by a rise in the marginal worker population share. Interestingly, marginal workers tend to consume more in compensation for a rise in welfare costs from working: average consumption for marginal workers increases. Not surprisingly, they also increase savings in terms of money holdings. However, non-marginal workers decrease consumption and saving. This implies that, from the general equilibrium perspective, these behaviors of marginal workers lead to a rise in the price level by increasing demand for money.

To sum up, in response to an unexpected rise in money supply, the population of marginal workers marginal rises, and they tend to increase consumption and savings more than others. This result leads to an important implication for the relation between the effectiveness monetary policy and inequality. Since marginal workers behave differently from other households, a general equilibrium effect depends on the distribution of marginal workers.[36] An economy

[35]The responses show level changes and the size of responses is normalized by the population share.

[36]The distribution of marginal workers affects the population size of these households in the economy and the gap between the values functions, $\left|V^E(\theta, \Theta, \widetilde{W}) - V^N(\theta, \Theta, W^c)\right|$ or $\left|V^N(\theta, \Theta, \widetilde{W}) - V^E(\theta, \Theta, W^c)\right|$.

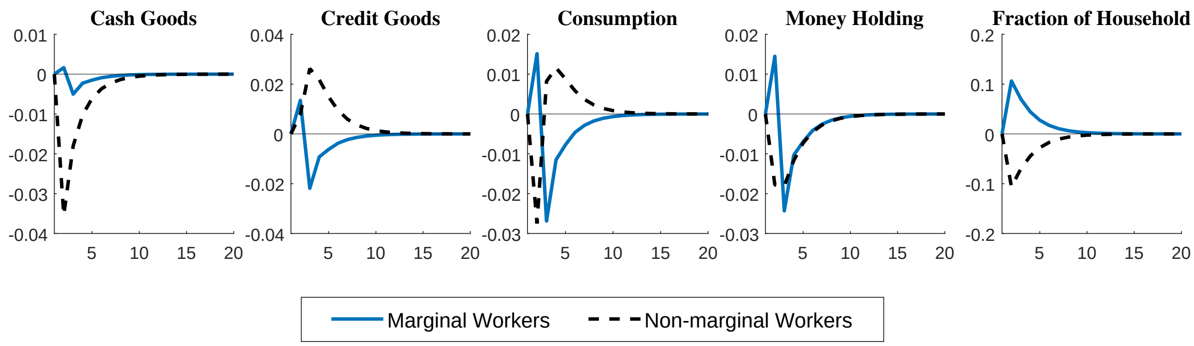


Figure 4.4: Behaviors of Marginal Workers

Note: The figure shows the responses of key per capita variables for marginal workers and non-marginal workers in response to an unexpected increase in money supply. Marginal workers are defined in Equation 4.8 and 4.9. The responses show level changes and the size of responses is normalized by the population share.

with a different level of inequality has a different marginal worker distribution, and therefore the effectiveness monetary policy will be different. I will discuss this issue in detailed in Section 5.

4.4.5 Distributional Effects of Monetary Policy

4.4.5.1 Reservation Wages Distribution

It is also useful to discuss the reservation wage distribution generated by the model economy to have better understanding of the employment decisions for households in the model economy, which is one of the key mechanism for monetary policy shocks. Figure 4.5 shows the reservation wage rate (per effective labor), $W^R(\theta, \Theta)$, distribution over labor productivity and net wealth in the model economy. As briefly discussed earlier, the reservation wage rate is different across households types: as expected, it is an increasing function of net asset holdings but is a decreasing function of labor productivity.

From this figure, we can have two implications. First, as discussed in Chang and Kim (2006), the aggregate labor supply responses depend on how different are individual reservation wages compared to the market wage. In other words, a marginal worker distribution

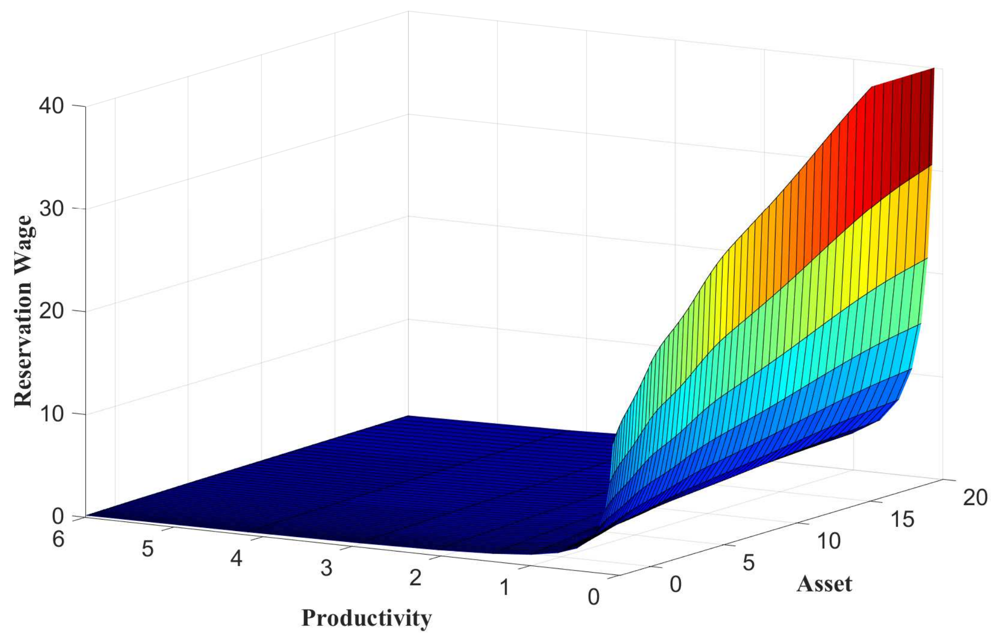


Figure 4.5: Reservation Wage Rate (per effective labor) Distribution

Note: The figure shows the per effective reservation wage distribution over productivity and assets.

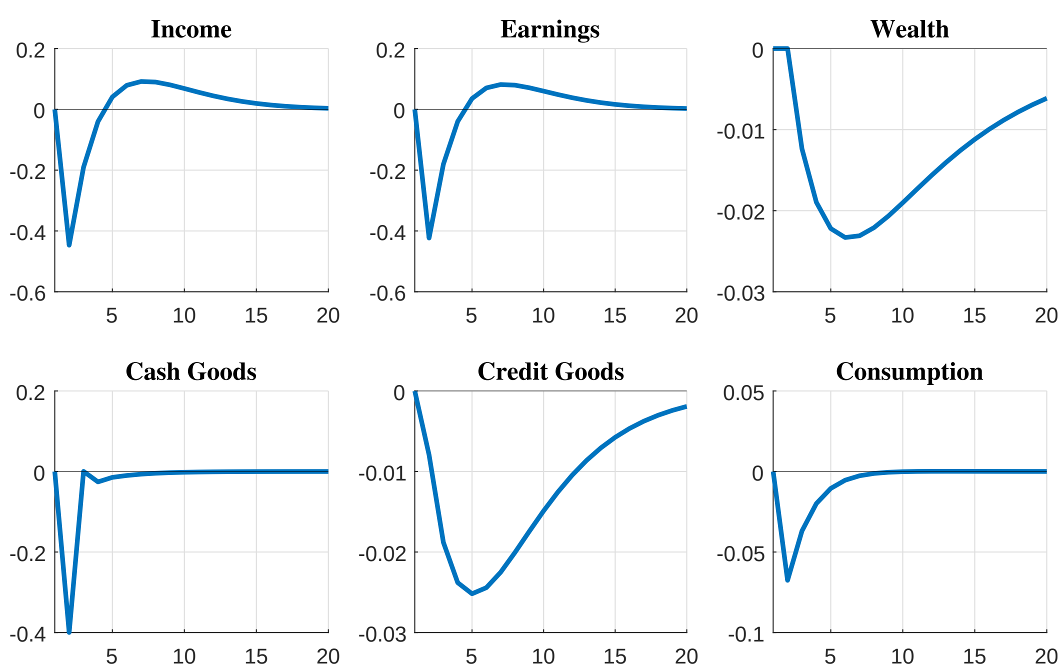


Figure 4.6: The Effects of Monetary Policy on Gini Coefficients

Note: The figure depicts the effects of one-standard-deviation expansionary monetary policy shocks on Gini coefficients of income, earnings, wealth, and consumption (cash goods, credit goods, and total consumption). The Gini coefficients are logged, so the responses are percentage deviations from the steady state.

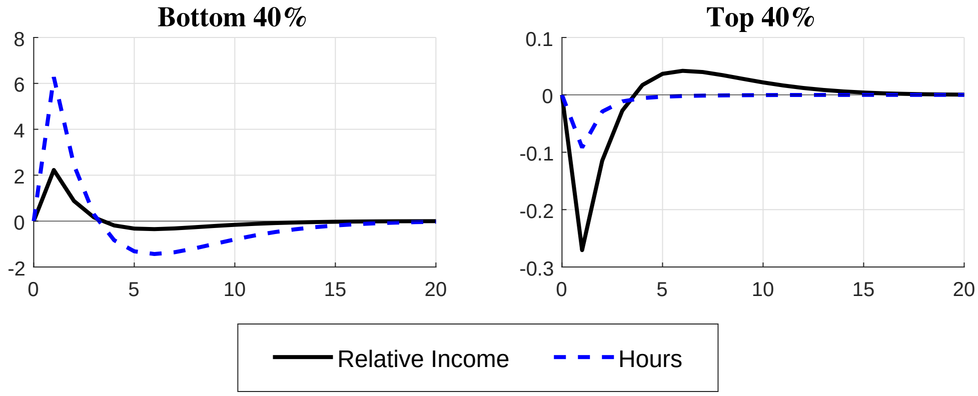


Figure 4.7: Responses of Relative Income and Hours over Income

Note: The figure shows responses of relative income to average and hours between the bottom and the top 40 percent of income.

can be determined by the reservation wage distribution. Since the shape of reservation wage distribution will depend on productivity and wealth distributions, a marginal worker distribution also will rely on the inequality status of the economy. I will discuss this issue in detail in Section 5.

Second, by assumption, households are employed in an ascending order by their reservation wage rates (per effective labor) until they provide what the firms demand. Hence, households whose reservation wage rates were slightly higher than the contract wage are likely to be newly employed in response to monetary policy shocks. In other words, since the reservation wage is a decreasing function of productivity and an increasing function of assets, employment of households with relatively low productivity or higher wealth will be mostly affected by changes in monetary policy actions. This implies that monetary policy can have substantially different effects on households in the economy, depending on their reservation wages. The distributional effects of monetary policy shocks will be discussed next.

4.4.5.2 The Effect of Monetary Policy on Inequality

One of the main focuses of this study is on the distributional effects of monetary policy shocks. Substantially different reservation wages across households suggest that monetary policy shocks could have sizable effects on inequality in the economy. Figure 4.6 depicts the effects of one-standard-deviation expansionary monetary policy shocks on Gini coefficients of income, earnings, wealth, and consumption (cash goods, credit goods, and total consumption). An expansionary monetary policy shock, indeed, decreases inequality in wealth, earnings, income, and consumption, as is suggested by the empirical literature such as Furceri, Loungani and Zdzienicka (2016) and Coibion et al. (2017). A one-standard-deviation expansionary monetary policy (money growth) shock reduces Gini coefficients of earnings, income, and consumption by 0.57, 0.54, and 0.07 percent, respectively. Since wealth is a state variable, it does not respond immediately, but its inequality decreases slowly after a monetary policy shock: the Gini coefficient of wealth falls by 0.03 percent after 5 quarters of the shock. Overall, the transmission channel through which an unanticipated monetary easing affects inequality in the model economy is through an increase in employment from the bottom of distributions induced by nominal wage contracts. Firms hire more workers after a monetary expansion due to a fall in real wages. As discussed above, newly employed households tend to be less productive. This implies that employment is likely to arise at the bottom of distributions since less productive households tend to belong to the bottom. This generally holds for income, earnings and consumption distributions since labor income is the main source of total income and consumption dynamics is largely affected by changes in income. Regarding the wealth distribution, one may argue that an unexpected increase in money supply would increase wealth inequality since newly employed households may be wealthy in that the reservation wage is also an increasing function of wealth. However, if new workers in the top of wealth distribution have relatively smaller productivity than the newly employed households in the bottom, wealth inequality can decrease after the monetary easing. In this model economy, these two effects indeed coexists, but wealth inequality falls as the latter effect dominates.

As an example of the distributional effects of monetary policy shocks, I show the responses

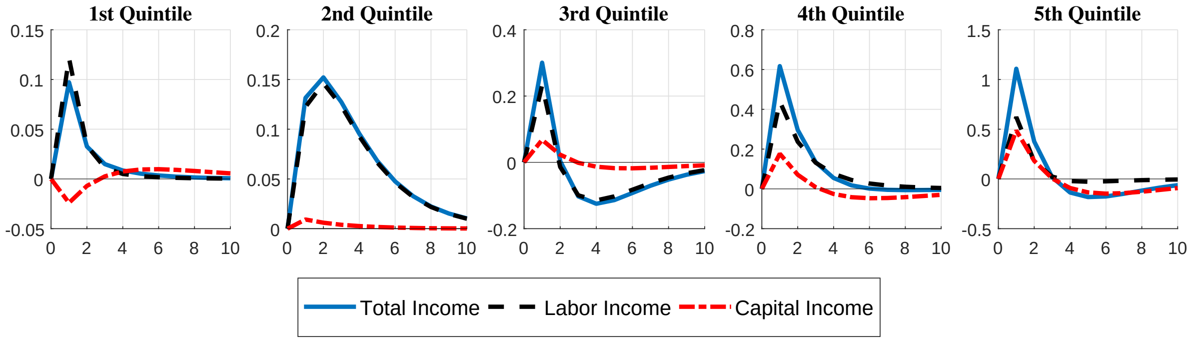


Figure 4.8: Decomposition of Income Share Responses

Note: The figure shows changes in levels of total income, labor income, and capital income to expansionary monetary policy shocks.

of relative income to average and hours over the income distribution in Figure 4.7. As explained above, income-poor households tend to be newly employed in response to an unexpected rise in money supply: hours at the bottom 40 percent increase by 6 percent. The increase in employment from the bottom of the income distribution leads to a fall in income inequality: the relative income for the poor (the bottom 40 percent) increases while the rich (the top 40 percent) lose their income in a relative sense, which makes the income Gini coefficient decrease by around 0.4 percent on impact.

Another interesting implication of the distributional effects of monetary policy is from the different responses of real wages and real asset prices. The former increases employment and in turn labor income while the latter leads to a rise in capital income. Hence, the different responses of factor rental prices result in asymmetric effects on income, depending on how households compose their income. To better understand how the different responses of real wages and real asset prices have compositional impacts, I decompose the responses of the level of income across the wealth distribution into two income sources, labor and capital incomes, in Figure 4.8 and Table 4.5. As shown in Figure 4.8, the increase in total income for the poor is mainly due to a huge rise in labor income which is induced by an increase in employment. For example, as found in Table 4.5, an increase in income on impact for households in the second wealth group is mostly explained by a rise in labor income: the

Table 4.5: Decomposition of Income Responses on Impact

	Quintile				
	1st	2nd	3rd	4th	5th
	Level Change				
Total Income	0.10	0.13	0.30	0.62	1.11
Labor Income	0.12	0.12	0.23	0.44	0.62
Capital Income	-0.02	0.01	0.07	0.18	0.49
	Contribution Rate (%)				
Total Income	100.0	100.0	100.0	100.0	100.0
Labor Income	124.59	93.01	77.43	71.32	56.10
Relative to mean	1.50	1.08	0.94	0.82	0.66
Capital Income	-24.59	6.99	22.57	28.68	43.90
Relative to mean	-1.86	0.53	1.37	2.04	2.92

Note: The upper panel shows a income level change to expansionary monetary policy shocks on impact, and the bottom panel shows contribution rates of each income sources to the total income change.

contribution rate of labor income to the rise in total income is 93 percent while it is only 7 percent for capital income. Relative contribution rates for labor income at the bottom of the wealth distribution is relatively large: it is 1.5 for the poorest wealth group but is only 0.66 at the top. This means that labor income of asset-poor households is relatively important when comparing it to their capital income and to labor income in other asset-rich groups. Interestingly, since an unexpected monetary expansion positively affects the real asset returns, capital income considerably rises for richer households. According to Table 4.5, the contribution of capital income increases with the level of asset holdings. For example, the contribution rate of capital income to the rise in total income is 44 percent for the top wealth group, which is three times as large as the mean. This implies that a rise in real asset return sharply increases capital income in the upper wealth quintiles. Therefore, there are compositional effects of monetary policy on income across the wealth distribution: the poor benefit from an increase in employment whereas the rich benefit from a rise in the real asset returns in a relative sense.

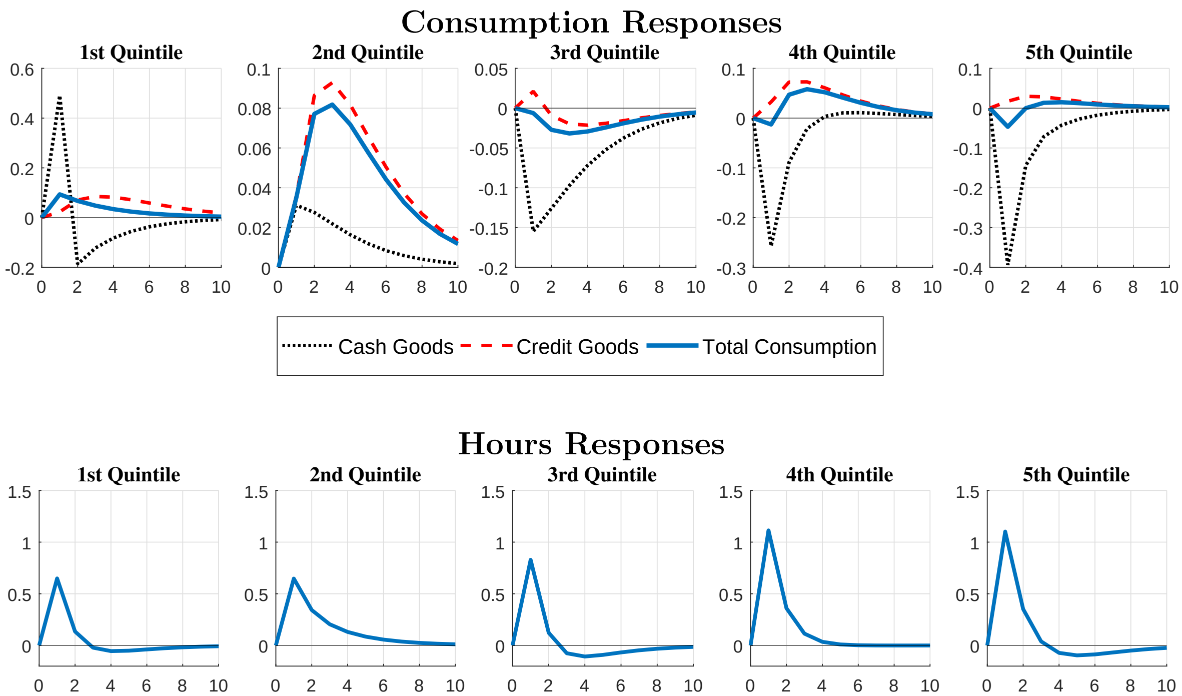


Figure 4.9: Responses of Consumption and Hours across Wealth Quintiles

Note: The upper panel depicts the responses of cash goods, credit goods, and total consumption to expansionary monetary policy shocks across the wealth quintiles, and the bottom shows the responses of hours across the wealth quintiles.

Importantly, there are asymmetric responses of consumption across the wealth distribution. The upper panel in Figure 4.9 exhibits the responses of cash goods, credit goods, and total consumption by the wealth quintiles to an unexpected rise in money supply. As far as cash goods are concerned, a rise in the price level hurts asset-rich households in the economy: households in the upper wealth quintiles (third to fifth wealth groups) reduce cash goods. This is because higher inflation plays a role in a rise in consumption taxes, and in particular, the wealthiest reduce cash goods significantly relative to other households. Households in the first and second wealth groups increase their cash goods since a sudden rise in the money transfer injected by the government dominates the effect caused by price level increase. Since all the wealth quintiles supply more hours to the firms as found in the bottom panel of Figure 4.9, their labor income rises,[37] and this rise allows households to increase their credit

[37] Of course, a reduction in real wages can decrease labor income to some extents, but a rise in hours may dominate this effect.

goods on average. On net, households in the bottom of the wealth distribution increase total consumption (the sum of cash and credit goods) while asset-rich households tend to decrease total consumption. Therefore, inflation hurts wealthy households more. This model result may support the empirical findings in the literature (e.g., Doepke and Schneider (2006)).

4.4.6 Empirical Evidence

I provide empirical evidence for the key mechanism or results in the model economy. I first discuss an empirical analysis on the cyclicality of real wages and real asset prices in response to monetary policy shocks. I then empirically document the heterogeneous responses of consumption and employment.

4.4.6.1 Responses of Real Wages and Real Asset Prices

In the model economy, sluggish adjustment of real wages is central for understanding the monetary transmission mechanism. Also, a rise in real asset returns is important in explaining the effects on asset-rich households. To estimate the response of real wages or real asset returns to monetary policy shock, I consider a three-variable Vector Autoregressive (VAR) model, including the measure of monetary policy shocks,[38] federal funds rates, and real wages or real asset prices. I consider the following reduced-form VAR,

$$Y_t = A(L)Y_{t-1} + v_t, \tag{4.10}$$

where Y_t is a vector including the monetary policy shock measure, federal funds rates, and logged real wages or real asset prices, $A(L)$ is a polynomial in the lag operator; $v_t \sim N(0, \Omega)$ are reduced-form innovations. The structural innovations, $\xi_t \sim N(0, I)$, are defined by an orthonormal rotation of the reduced-form residuals:

[38] Measures for monetary policy shocks are from Coibion et al. (2017). They update the measure of Romer and Romer (2004) until the fourth quarter of 2008.

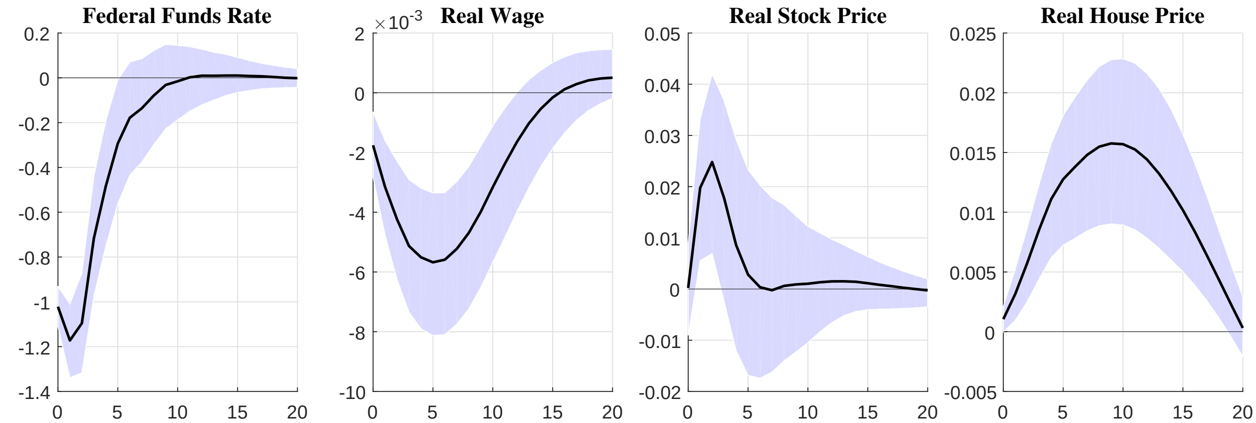


Figure 4.10: Responses of Real Wage and Real Asset Prices

Note: The figure shows the responses of real wages and real asset prices to an expansionary monetary policy shock. Real wages are defined as average hourly earnings of production and nonsupervisory employees for total private deflated by producer price index for all commodities; real house prices are real residential property prices; real stock prices are defined as total share prices for all shares deflated by price index for all urban consumers for all items.

$$\xi_t = B_0 v_t, \tag{4.11}$$

where $B_0^{-1}(B_0^{-1})' = \Omega$. I order the measures for monetary policy shocks first and real wages or real asset prices last in the VAR and identify the matrix B_0^{-1} using the Cholesky decomposition of Ω. Constant terms, fourth-degree-polynomial trend terms, and four lags are included in the VAR. As discussed in Anderson, Inoue and Rossi (2016), by including the shocks measures in a VAR where the shock is ordered first, we can ensure that the shock is uncorrelated with past information contained in the other variables in the VAR, and other variables response to the shocks contemporaneously. Real wages are defined as average hourly earnings of production and nonsupervisory employees for total private.[39] Real house price is real residential property prices, and real stock price are defined total share prices for all shares.[40]

[39]It is deflated by producer price index for all commodities

[40]Real stock prices are computed by deflating them by price index for all urban consumers for all items

The responses of real wages and real asset prices to an expansionary monetary policy shock are reported in Figure 4.10. I first discuss the dynamic behavior of real wages. As expected by the model economy, the real wage drops after a monetary easing as shown in the second column. The real wage rate decreases on impact and has a smallest value in 5 quarters after the shock. This empirical result is inconsistent with that in Christiano, Eichenbaum and Evans (2005), who find that the real wage is mildly procyclical in response to monetary policy shocks.[41] However, there are also abundant empirical evidence suggesting that real wage is coutercyclical in response to monetary policy shocks. For example, Spencer (1998) provides empirical findings that the real wage response is strongly and robustly negative in response to demand shocks, which supports sticky-wage theories of the business cycle including the model economy in this paper. Similarly, Balmaseda, Dolado and Lopez-Salido (2000) empirically document that real wages are countercyclical in response to aggregate demand shocks. Leiderman (1983) also finds that real wage response to an unanticipated increase in money growth is weakly negative.[42]

As discussed by Kaplan, Moll and Violante (2018), having asset price movements consistent with empirical evidence is potentially important in understanding the transmission mechanism of monetary policy. Werning (2015) also shows that the response of consumption to monetary policy shock depends on the cyclical movement of asset prices. In light of this discussion in the literature, cyclical real asset returns generated in the model economy are well supported by empirical evidence. The third and fourth columns show the responses of real stock prices and real house prices, respectively, to an unanticipated monetary easing. Consistent with the model assumptions, both real asset prices increase with an unexpected monetary expansion. The response of the real house price is smoother that of the real stock

[41] Gamber and Joutz (1993) also find in general a positive dynamic response of real wages to monetary policy shocks.

[42] One possible reason why there is no broad agreement among researchers on the basic empirical facts regarding wage rigidity is that real wages are hard to measure, and it is difficult to know whether observed wages are allocative (Basu and House, 2016). Regarding this issue, Lastrapes (2002) concludes that the reason for the disaggrement is a lack of robustness of the estimated wage response functions to model specification, data transformation to induce stationarity, the choice of proxy for the aggregate real wage, and the choice of variables to include in the VAR.

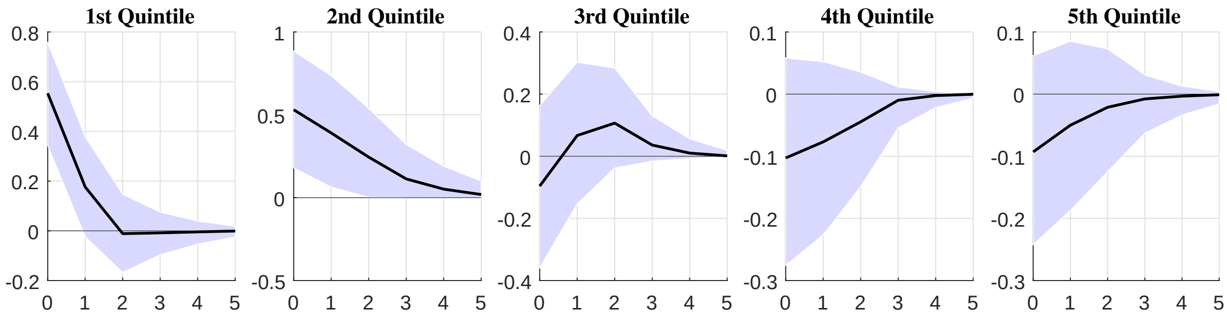


Figure 4.11: Response of Employment across Income Quintiles

Note: The figure presents the responses of employment across different income quintiles to an expansionary monetary policy shock.

price, which suggests that there are more frictions in the housing market. Empirical studies (e.g., Furceri, Loungani and Zdzienicka (2016)) in the literature also find the consistent results.

To sum up, the dynamic behaviors of both the real wages and real asset prices in the model economy are well supported by the empirical results.

4.4.6.2 Heterogeneous Responses of Employment

I also provide supportive empirical evidence for the different responses of employment across income groups to monetary policy shocks, which is the key channel of the model. For this analysis, I use the Current Population Survey (CPS). Since information on income in the CPS is available in March of every year, I cannot use quarterly data for employment rates across income quintiles. Hence, I use annual data spanning 1969 to 2008. To estimate the effects of monetary policy shocks on employment by income, I employ the VAR where the monetary policy shocks measure are ordered first, and constant terms, quadratic trend terms, and a one-period lag are included. Figure 4.11 reports the estimated response of employment rates across income groups. As predicted by the model economy, households in the lower income quintiles increase employment while employment rates for richer households tend to decrease in response to expansionary monetary policy shocks.

4.4.6.3 Heterogeneous Responses of Consumption

I also provide empirical evidence that there are heterogeneous responses of consumption to monetary policy shocks. To this end, I use the Consumer Expenditure Surveys (CEX), which is conducted by the Bureau of Labor Statistics (BLS), to collect information on consumption and age across individual households.[43] I mainly use quarterly data which span from the first quarter of 1980 to the fourth quarter of 2008. The measure of non-durable consumption includes food and beverages, tobacco, apparel and services, personal care, gasoline, public transportation, household operation, medical care, entertainment, reading material and education. The definition of the non-durable goods is similar to that of Anderson, Inoue and Rossi (2016) and De Giorgi and Gambetti (2012).[44] Non-durable consumption for households is real per capita values: they are divided by family size (the number of family members), deflated by CPI-U series, and seasonally adjusted by X-12-ARIMA. Since net wealth data are not available not even for every year, I use age dimension as a proxy for asset holdings. The measure of age is defined as the age of a head of each household.

I order the measures for monetary policy shocks first, federal funds rates next, and consumption for each age group last in the VAR. Figure 4.12 presents the responses of consumption across different age groups to an expansionary monetary policy shock. The model economy predicts that households in the bottom of the wealth distribution increase total consumption while asset-rich households tend to decrease total consumption. As shown in Figure 4.12, the first age group increases their consumption and older households (the fourth and fifth quintiles) tend to reduce their consumption in response to an expansionary monetary policy shock. Considering that older households are asset-rich while the young cohorts own relatively less amount of wealth, the empirical result regarding the heterogeneous

[43]The CEX is rotating panel data where individuals are interviewed for four consecutive quarters at most.

[44]Anderson, Inoue and Rossi (2016) define non-durable consumption as expenditures on food, alcoholic beverages, tobacco, utilities, personal care, household operations, public transportation, gas and motor oil, and miscellaneous expense, and De Giorgi and Gambetti (2012) use food (including alcohol and tobacco), heating fuel, public and private transport (including gasoline), and personal care as the non-durable consumption.

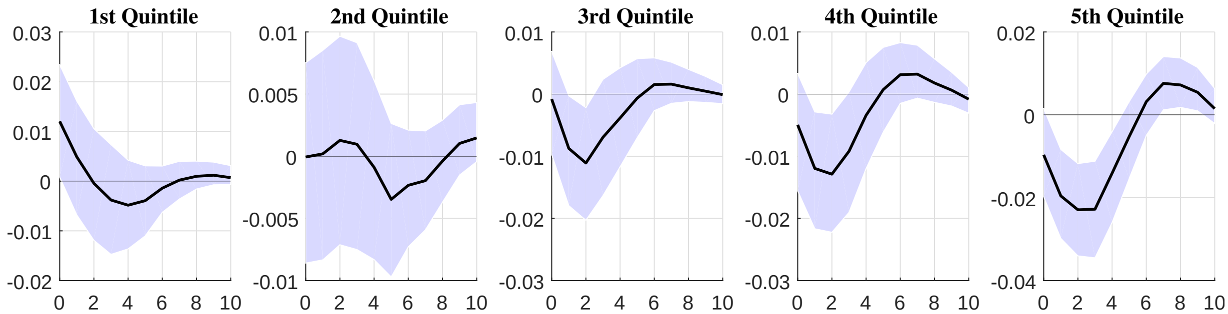


Figure 4.12: Response of Consumption across Age Quintiles

Note: The figure presents the responses of consumption across different age quintiles to an expansionary monetary policy shock. Consumption is defined as non-durable consumption, and data for consumption are from the CEX 1980:Q1−2008:Q3.

behaviors of consumption across the age distribution is broadly consistent with the model predictions.

This empirical finding is consistent with Doepke and Schneider (2006) and Wong (2018). Doepke and Schneider (2006) quantitatively assess the effects of inflation through changes in the value of nominal assets and find that rich and old households are main losers from inflation. Similarly, Wong (2018) empirically estimates age-specific consumption responses to monetary policy shocks and finds that consumption of younger people is more responsive.

4.5 The Impact of Inequality on the Effectiveness of Monetary Policy

I next turn to the other main contribution of this paper: the role of the long-run level of inequality in the effectiveness of monetary policy. What is the main linkage for the relation? To give a concrete answer to this question, I will take three steps. I first demonstrate the relation between labor supply elasticity and the monetary policy effectiveness using the representative-agent model. I then discuss how the long-run level of inequality determines the labor supply elasticity in the model economy where indivisible labor is combined with heterogeneous households. Finally, I integrate the long-run level of inequality and the effectiveness of monetary policy by using labor supply elasticity as the main linkage for the relation.

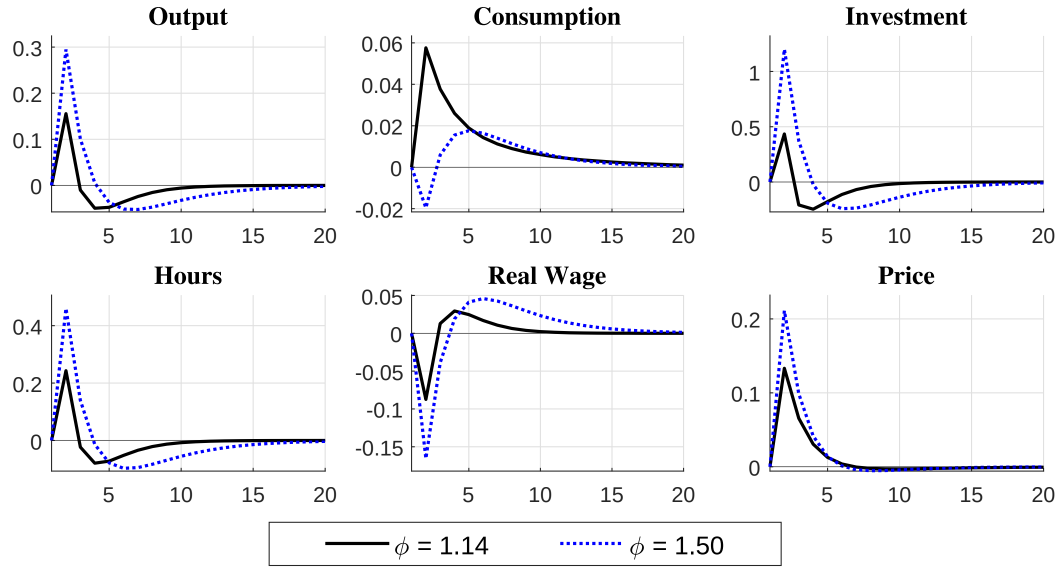


Figure 4.13: Response of Key Variables for Different ϕ

Note: The figure shows the responses of key aggregate variables of two representative-agent economies with different values of ϕ ($\phi = 1.14$ and $\phi = 1.5$) to expansionary monetary policy shocks.

4.5.1 Labor Supply Elasticity and Monetary Policy Effectiveness

Given the utility function, ϕ represents the elasticity of intertemporal substitution of labor supply or Frisch elasticity of labor supply.[45] Hence, hours worked supplied by households will be more sensitive to changes to wages with a larger value of ϕ. As discussed by Cho, Cooley and Phaneuf (1997), the parameter ϕ also captures the attitude of households toward risk with respect to labor supply: households are less risk averse with respect to hours worked with a high value of ϕ.[46] Since the wage contract generates a higher variation in labor supply, and a degree of risk aversion with regard to labor supply depends on the labor supply elasticity parameter, ϕ, households behave differently with different values of ϕ.

Based on the representative-agent models with nominal wage contracts, I demonstrate the relation between labor supply elasticity and the monetary policy effectiveness. The responses of key aggregate variables of two representative-agent economies with different

[45]The Frish elasticity captures the elasticity of hours worked to the wage rate for a constant marginal utility of wealth. Hence, it measures a pure substitution effect of a wage change.

[46]Cho, Cooley and Phaneuf (1997) find that the welfare cost of nominal wage contract decreases as the value of ϕ gets larger.

values of ϕ to monetary policy shocks are shown in Figure 4.13. As expected, a degree of risk aversion with regard to labor supply matters for the effectiveness of monetary policy. According to Figure 4.13, an economy with higher ϕ has more effective monetary policy in terms of output. In response to a one-standard-deviation monetary policy (money growth) shock, output increases by 0.15 percent when $\phi = 1.14$ while it rises by 0.3 percent in the economy where $\phi = 1.5$. The main mechanism of the larger effectiveness of comes from the interplay between a degree of risk aversion with regard to labor supply and the nominal wage contract. Under the wage contract, households must work more than the equilibrium level with an expansionary monetary shocks, which leads to a a decrease in welfare. With a large degree of risk aversion or a small value of ϕ, households desire to consume more in compensation for an increase in disutility caused by the wage contract, which implies that save less. Since households save by accumulation assets and holding money, demand for money increases by less than in the economy with a smaller degree of risk aversion, which results in a smaller rise in price level. As found in Figure 4.13, the price increases by 0.13 percent when $\phi = 1.14$ while it rises by 0.21 percent when $\phi = 1.5$. A rise in price level has the general equilibrium effect: it decreases real wages, and firms hire more workers. This general equilibrium effect is larger in the economy with a smaller degree of risk aversion or a larger value of ϕ.

4.5.2 Inequality and Labor Supply Elasticity

As found in Chang and Kim (2006) and Rogerson and Wallenius (2009), a degree of heterogeneity in an economy would affect aggregate labor supply elasticity. To be more specific, reservation wages for households can be computed due to the indivisible labor supply assumption in the heterogeneous-agent model. Using the reserve wages across households, the model can endogenously generate different aggregate labor supply curves across the different level of heterogeneity. Intuitively, less heterogeneity is associated with larger aggregate labor supply elasticity. The mass of marginal workers[47] matters will determine macro labor

[47] "A marginal worker" reservation wage is close to the market wage.

supply elasticity in the economy. In the more equal economy, there would be more marginal workers, which implies that labor supply elasticity in this society would be relatively large. In other words, labor supply is more elastic for an economy with a small heterogeneity as the reservation wage distribution is more concentrated.

To investigate the role on heterogeneity in aggregate labor supply elasticity, I consider the model economy with less heterogeneity by decreasing the standard deviation of the idiosyncratic shock process, σ_x, to 0.230, which is 20 percent less than the benchmark value. In the economy with less heterogeneity, Gini index for income and wealth reduce to 0.53 and 0.65, respectively, which are 0.59 and 0.68 in the benchmark economy ($\sigma_x = 0.287$). The reservation-wage schedule and invariant distribution allows us to uncover the aggregate labor-supply curve of the economy. Figure 4.14 reports the inverse cumulative distributions of reservation wages with corresponding participation rates for two different model economies: the economies where $\sigma_x = 0.287$ and $\sigma_x = 0.230$. The solid lines represent the benchmark economy ($\sigma_x = 0.287$) while the dotted lines show a counterfactual economy with low inequality ($\sigma_x = 0.230$). Based on these reservation wage schedules, the responsiveness of labor market participation can be computed. I calculate the elasticity at employment rates of 60 percent, which is the steady state employment rate in both economies. For the benchmark economy ($\sigma_x = 0.287$), the elasticity is 1.14 at the steady-state employment rate, while it is 1.5 in the economy with a smaller degree of heterogeneity ($\sigma_x = 0.230$). Hence, a more equal economy has a larger degree of labor supply elasticity, which implies that an economy with less heterogeneity has a less disperse distribution of reservation wages.

4.5.3 The Role of Inequality in the Monetary Policy Effectiveness

Next, I investigate the role of long-run level of inequality in the effectiveness of monetary policy. The main linkage between the long-run level of inequality and the effectiveness of monetary policy is aggregate labor supply elasticity (or the shape of reservation wage distribution). The intuitive explanation is as follows. Suppose that two economies with different heterogeneities have a drop in real wages by the same amount in response to expansionary

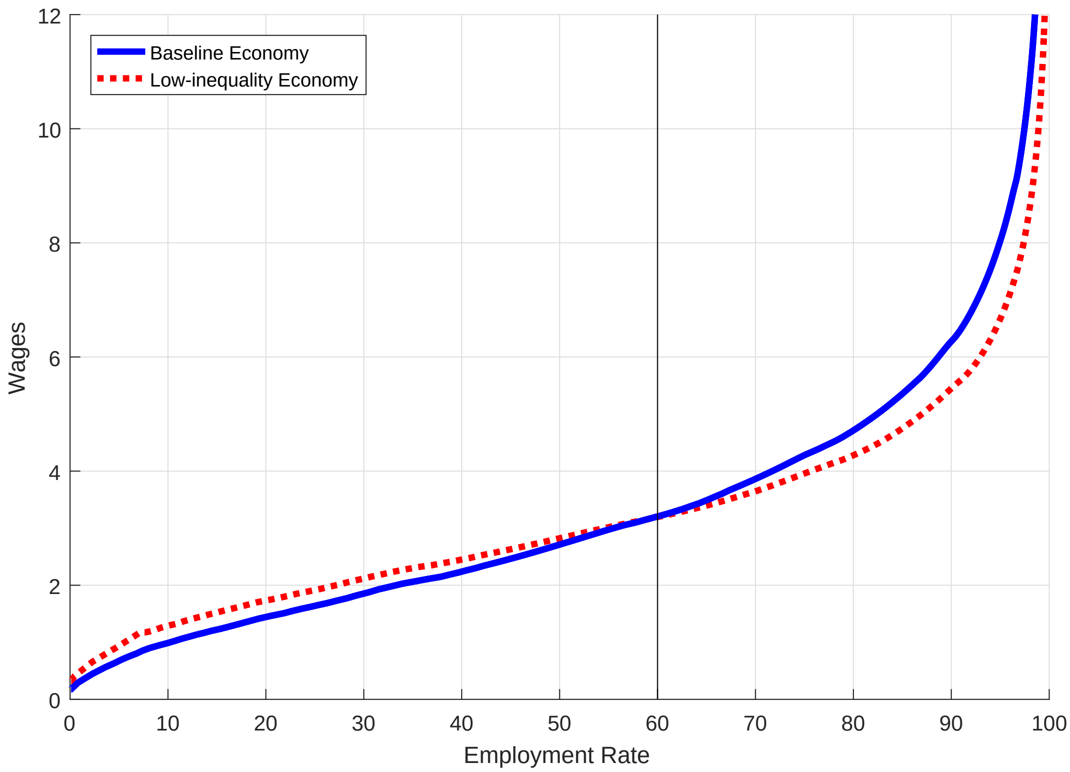


Figure 4.14: Labor Supply Curves with Different Heterogeneity

Note: The figure shows the inverse cumulative distributions of reservation wages with corresponding participation rates for two different model economies: the economies where $\sigma_x = 0.287$ and $\sigma_x = 0.230$. The solid lines represent the benchmark economy ($\sigma_x = 0.287$) while the dotted lines show a counterfactual economy with low inequality ($\sigma_x = 0.230$).

monetary policy. As discussed earlier, a decrease in real factor prices for labor services allows the firms to hire more workers, and households are employed in ascending order by their reservation wage rates (per effective labor) until they provide the effective labor the firms demand. Since a size of responsiveness of real wages is assumed to be identical, the effective labor that the representative firm demands will be the same for the two economies. However, decisions of credit goods will be different in the two economies. Larger labor supply elasticity or a more concentrated reservation wage distribution means that a welfare difference between employment and nonemployment is relatively small, and in turn households tend to increase credit goods by less since disutility from involuntarily working is relatively small. This implies that households in a low-inequality economy increase credit goods by less, and this will lead to a rise in money demand as means of savings. An increase in money demand results in a rise in price and a fall in real wages. Finally, labor demand will rise by more, and this leads output to increase by more in the economy with less heterogeneity.

Figure 4.15 shows the responses of key aggregate variables of two model economies to monetary policy shocks: the economies where $\sigma_x = 0.287$ and $\sigma_x = 0.230$. As expected, the long-run level of inequality matters for the effectiveness of monetary policy. According to Figure 4.15, a more equal society is associated with more effective monetary policy in terms of output, *ceteris paribus*. Output increases more in the model economy with less heterogeneity that those in the benchmark case: it increases by around 20 percent more than that in the benchmark counterpart. As explained earlier, the main mechanism of the larger effectiveness of comes from the shape of reservation wage distribution, which is represented by the labor supply elasticity. A rise in money demand is larger in the model with less heterogeneity since households in the low-inequality economy reduce their credit goods increase by less due to smaller welfare costs induced by involuntary employment. This generates a larger increase in the price level in the economy as shown in Figure 4.15. The higher inflation leads to a bigger drop in real rental rate for labor services, and finally it makes the firms to demand larger effective labor with more workers.

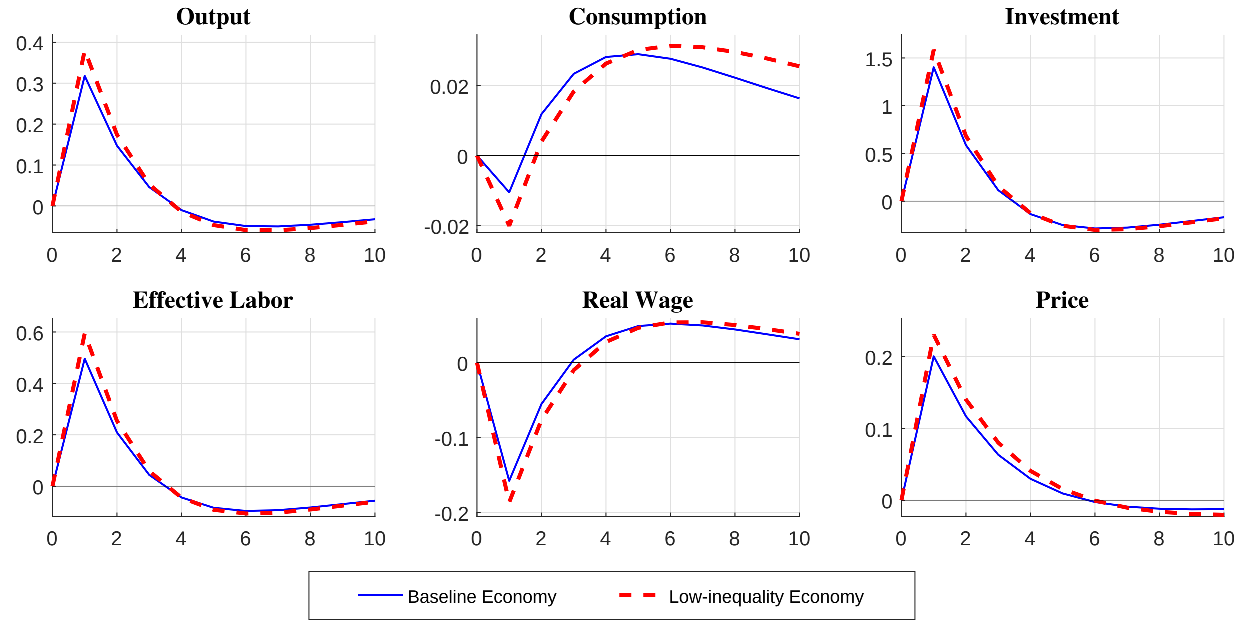


Figure 4.15: Responses of Key Variables with Different Heterogeneity

Note: The figure shows the responses of key aggregate variables of two model economies to monetary policy shocks: the economies where $\sigma_x = 0.287$ and $\sigma_x = 0.230$. The solid lines represent the benchmark economy ($\sigma_x = 0.287$) while the dotted lines show a counterfactual economy with low inequality ($\sigma_x = 0.230$).

To sum up, different heterogeneity in the model economy generates a different level of general equilibrium effects, and it finally affects the effectiveness of monetary policy: a more equal economy has a larger effectiveness of monetary policy in terms of output responses.

4.5.4 Empirical Evidence

In this section, I provide suggestive empirical evidence for the negative relation between long-run inequality and the monetary policy effectiveness, by using state-level data with national level policy variables.

4.5.4.1 Data

I use state-level data sets for inequality measures in the U.S. but consider national level shocks. By using state-level data with common shocks, I can address two potential problems that would occur in a country-level analysis: one is endogeneity problems due to considerable unobservable heterogeneity across countries, and the other is the normalization issue induced by different sizes of country-level monetary policy shocks. A state-level panel with national shocks in this study helps solve these two issues to some extent since there might be relatively less heterogeneity across states within a country, and common policy shocks are automatically normalized across states since each state faces the identical shocks.

Data on income inequality are taken from Frank (2014), who constructs the inequality measures by state using the pre-tax adjusted gross income published in the Internal Revenue Service (IRS). Frank (2014) computes various measures of income inequality, including the relative mean deviation, Gini coefficient, Atkinson index, Theil's entropy index, as well as the top 1% and top 10% income shares. Among them, Gini coefficients are mainly used for this study.

Regarding measures for monetary policy shocks, I use data constructed by Coibion et al. (2017). These measures are a combination of the use of Greenbook forecasts and narrative methods. Using narrative methods, Romer and Romer (2004) compute estimates of variations in intended federal funds rate during Federal Open Market Committee (FOMC) meetings. They then regress the intended federal funds rate on the output and inflation forecasts in the Greenbook at each FOMC meeting date and use the residuals as the monetary policy shocks. Based the method that Romer and Romer (2004) used, Coibion et al. (2017) extend the shock measures until 2008.

4.5.4.2 Estimation I: Two-step Regression

In order to estimate the impact of the long-run level of inequality on the effectiveness of monetary policy, I employ a two-step approach. In the first step, I consider a three-variable Vector Autoregressive (VAR) model including the monetary policy shock measure, federal fund rates, and GDP for each state and to obtain the effect of monetary policy on output for state i, denoted by δ_i. The second step is a cross-sectional estimation: I regress δ_i on a measure for inequality which are long-run averages across time for each state, i.

Figure 4.16 shows the relation between the long-run level of inequality and the effectiveness of monetary policy across states, where the effectiveness of monetary policy is measured by the peak output response to expansionary monetary policy shocks, and the long-run level

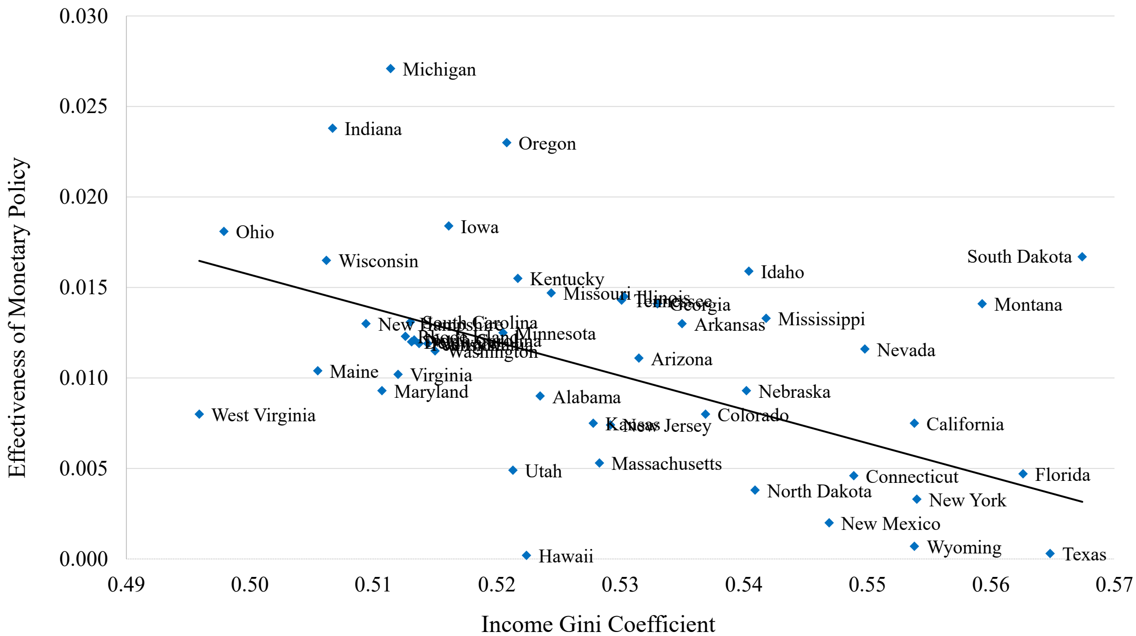


Figure 4.16: Inequality and the Effectiveness of Monetary Policy I

Note: The figure depicts the relation between the long-run level of inequality and the effectiveness of monetary policy across states, where the effectiveness of monetary policy is measured by the peak output response to expansionary monetary policy shocks, and the long-run level of inequality is defined as the average of historical Gini income coefficients.

of inequality is defined as the average value of historical Gini income coefficients. As shown in Figure 4.16, there is a negative relationship between inequality and the effectiveness of monetary policy. The simple correlation between the two variables is -0.5. I regress the effectiveness of monetary policy on the long-run level of inequality. According to Table 4.6, the slop coefficient is negative and statistically significant: it is -0.18 and is significant at the one percent level. This result is still robust even when I include the long-run level of GDP per capita as a control variable following Brinca et al. (2016).

4.5.4.3 Estimation II: Local Projection Method with Dummy

As a robustness check, I also use local projection method proposed by Jorda (2005) to estimate impulse response functions (IRFs).[48] Local projection method has been increasingly used in applied work and can easily be extended to estimate state-dependent effectiveness of

[48]Ramey and Zubairy (2018) use local projection method to estimate impulse responses and government spending multipliers in the U.S., and Furceri, Loungani and Zdzienicka (2016) also study the effect of monetary policy shocks on income inequality using the local projection method with a panel of 32 countries.

Table 4.6: Inequality and the Effectiveness of Monetary Policy I

Dependent Variable: Effectiveness of Monetary Policy		
	(1)	(2)
Constant	0.1067***	0.2221***
	(0.0325)	(0.0609)
Gini Coefficient	-0.1824***	-0.1658***
	(0.0619)	(0.0540)
GDP Per Capita		-0.0122**
		(0.0053)
Observation	50	50
R-squared	0.23	0.34

Note: The table shows the regression results for the relation between the long-run level of inequality and the effectiveness of monetary policy across states, where the effectiveness of monetary policy is measured by the peak output response to expansionary monetary policy shocks, and the long-run level of inequality is defined as the average of historical income Gini coefficients. Values in ( ) are White's robust standard errors. ** and *** indicate significance at the 5% and 1% levels, respectively. Per-capita GDPs are logged values.

economic policies (e.g., **?**). To be more specific, the local projection method for panel data simply estimates a series of regressions for each horizon j:

$$y_{i,t+j} = \alpha_{i,j} + \mathbb{I}^H_{i,t-1}\beta_{H,j}[Shock_t + \gamma_{H,j}x_{i,t}] + \mathbb{I}^L_{i,t-1}\beta_{L,j}[Shock_t + \gamma_{L,j}x_{i,t}] + \varepsilon_{i,t+j} \tag{4.12}$$

where $j = 0, 2, ..., J$, y is logged GDP, α is state fixed effects which can control for unobserved cross-state heterogeneity, $Shock$ is monetary policy shocks, and x is a vector of control variables. x includes lagged policy shocks and lagged y. IRFs of y at time $t+j$ to the shock at time t can be obtained using the estimated coefficient $\beta_{H,j}$ and $\beta_{L,j}$. $\mathbb{I}^H_{i,t-1}(\mathbb{I}^L_{i,t-1})$ represents a high(low)-inequality state for each state i at time $t-1$. I use $\mathbb{I}^H_{i,t-1}$ in the regression rather than $\mathbb{I}^H_{i,t}$ since monetary policy at time t can affect inequality at time t. I set $\mathbb{I}^H_{i,t-1}(\mathbb{I}^L_{i,t-1}) = 1$ when a state i is one of the top (bottom) 10 states among 50 states in terms of the income Gini coefficient at time $t-1$ while $\mathbb{I}^H_{i,t-1}(\mathbb{I}^L_{i,t-1}) = 0$ otherwise. Hence, $\beta_{H,j}$

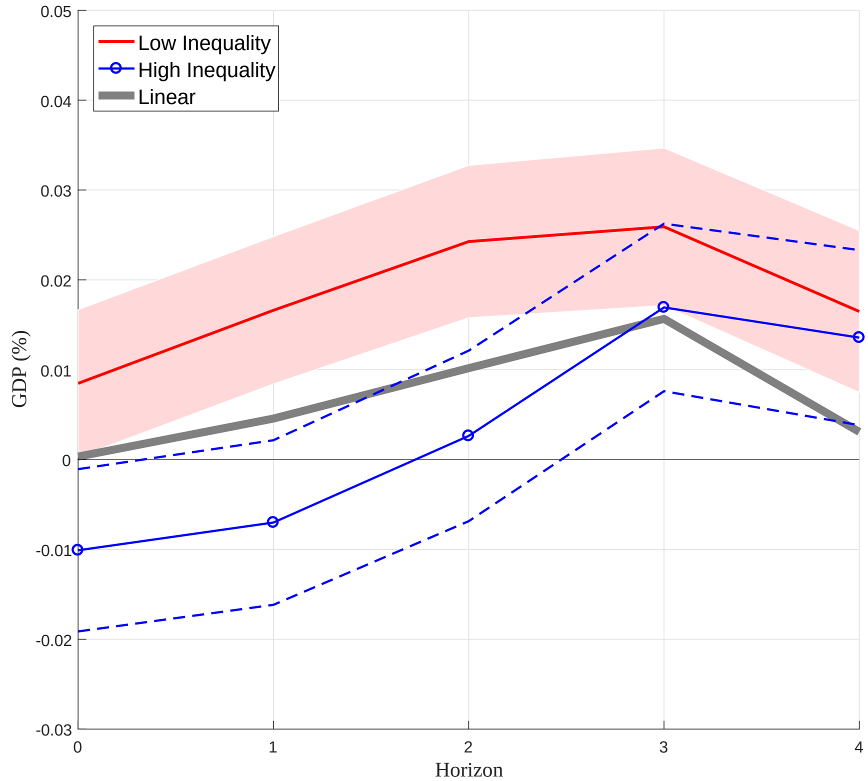


Figure 4.17: Inequality and the Effectiveness of Monetary Policy II

Note: The red line (the blue line with circles) shows the response of output to an expansionary monetary policy shock for the low (high) inequality states, and the gray thick solid line represents the unconditional response of output in the linear model. Newey-West standard errors are used, and 68% confidence intervals are shown with shaded area.

is the coefficient associated with the high-inequality states for a horizon j while $\beta_{L,j}$ captures the effects of the shock for low-inequality state for a horizon j. Two complications associated with the local projection method in the panel study is (i) covariance structures that vary by a certain characteristic and (ii) the serial correlation in the error terms induced by the successive leading of the dependent variable. In order to address the first issue, I have to use clustered robust standard errors, but the number of clusters is too small. Hence, I decide not to use clustered robust standard errors for the benchmark estimation, but the results with those error are also reported in the appendix and are still robust. For the second issue, I use standard errors with Newey-West correction, which is a standard correction method for the serial correlation problem in the literature.

The estimated responses of GDP to expansionary monetary policy shocks for states of high and low inequality are reported in Figure 4.17. The results of this exercise show that monetary policy shocks have much larger impacts on output during states of low inequality than high inequality. The effects of monetary policy shocks are negative on impact during states of high inequality, while the on-impact responses are positive when the economy is experiencing low inequality. There are statistically significant differences between the responses of output for high- and low-inequality states in one and two years after the shocks. Hence, the effects of expansionary monetary policy shocks on GDP in the linear estimation model (the thick solid line) are largely driven by low-inequality states. This empirical results provide supportive evidence that a more equal society is associated with more effective monetary policy in terms of output. The empirical finding is broadly consistent with those in the literature on the effectiveness of economic policy and inequality or indebtedness. Yang (2017) investigates the relationship between income inequality and the local government spending multipliers using rich historical state-level data on military procurement and inequality. He finds that the effects of government spending shocks on output are larger in low-inequality states than in high-inequality states. Alpanda and Zubairy (2017) also provide empirical evidence that the effects of monetary policy are less powerful during periods of high household debt. Similarly, Voinea, Lovin and Cojocaru (2018) find that lower inequality is associated with stronger effectiveness and higher homogeneity of monetary policy transmission.

4.6 Conclusion

This study investigates the relation between monetary policy and inequality by asking how one affects the other: the effect of monetary policy on inequality and the role of the long-run level of inequality in the effectiveness of monetary policy. To this end, I develop a simple dynamic stochastic general equilibrium (DSGE) model which incorporate nominal wage contracts and cash-in-advance constraints into a heterogeneous agent model economy with indivisible labor. Two main findings emerge: monetary policy affects inequality in the short run, and inequality matters for the effectiveness of monetary policy in the long run.

In regards to the first main finding, an unexpected monetary expansion decrease inequality. An expansionary monetary policy shocks leads to a fall in real wages due to the nominal wage contract, and this allows firms to hire more workers. A rise in employment from the bottom of distributions reduces inequality. There are heterogeneous effects of a monetary expansion on income and consumption across wealth distribution. Wealth-richer households benefit from a rise in the real asset returns while households at the bottom of the wealth distribution benefit from an increase in employment in a relative sense. Asset-poor households tend to increase consumption whereas consumption for wealthy households falls, which implies that inflation hurts rich households more.

As far as the second main finding is concerned, a more equal economy tends to have more effective monetary policy. The size of aggregate labor supply elasticity, which depends on a degree of heterogeneity in an economy, affects the responses of prices and real wages to monetary policy shocks. In an economy with less heterogeneity, the effects of monetary policy shocks on price and real wages will be larger, which leads to a bigger response of output.

5. SUMMARY AND CONCLUSIONS

This dissertation investigates inequality, business cycles, and macroeconomic policies.

First, I study the quantitative implications of real wage rigidities and heterogeneity for the two long-standing puzzles in the business cycles literature, the weak comovement of hours worked with labor productivity and the large cyclical movement in the labor wedge. I shed light on these issues by extending a heterogeneous-agent model with an indivisible labor supply choice to real wage rigidities.

The main findings of this paper can be summarized as follows. I find that the correlation coefficient between hours and productivity decreases, and the volatility of the labor wedge increases when the index of wage stickiness or the length of wage contracts increases. Heterogeneity also plays a role in solving the two puzzles since heterogeneity allows the aggregate labor supply curve to move in response to aggregate productivity shocks and the wedge to be endogenously produced. From these results, I argue that a small amount of real wage stickiness would be sufficient to resolve both anomalies when long-term wage contracts and heterogeneity are taken into account.

Second, I uncover why consumers behave differently in response to a government spending shock. To this end, I construct a heterogeneous agent model economy which incorporates a progressive taxation scheme, productive government expenditure, and indivisible labor. I find that the model economy successfully replicates the different responses of consumption between the bottom and the top of income distribution to government spending shocks. When the government increases its spending accompanied with a rise in tax progressivity, poor households are employed and hence increase their consumption due to an increase in after-tax wage rates while the rich decrease consumption since the effect of productive government spending cannot fully offset a significant increase in tax rates.

Existing theoretical macroeconomic models inspired by Gali, Lopez-Salido and Valles (2007) suggest that credit-constrained consumers are crucial to account for why government spending shocks have substantially different effects on consumers. On the other hand, this study proposes a new perspective by suggesting that it is important to consider different tax burdens across consumers when studying the distributional effects of government spending shocks.

Third, I investigate the relation between monetary policy and inequality by asking how one affects the other: the effect of monetary policy on inequality and the role of the long-run level of inequality in the effectiveness of monetary policy. To this end, I develop a simple dynamic stochastic general equilibrium (DSGE) model which incorporate nominal wage contracts and cash-in-advance constraints into a heterogeneous agent model economy with indivisible labor. Two main findings emerge: monetary policy affects inequality in the short run, and inequality matters for the effectiveness of monetary policy in the long run.

In regards to the first main finding, an unexpected monetary expansion decrease inequality. An expansionary monetary policy shocks leads to a fall in real wages due to the nominal wage contract, and this allows firms to hire more workers. A rise in employment from the bottom of distributions reduces inequality. There are heterogeneous effects of a monetary expansion on income and consumption across wealth distribution. Wealth-richer households benefit from a rise in the real asset returns while households at the bottom of the wealth distribution benefit from an increase in employment in a relative sense. Asset-poor households tend to increase consumption whereas consumption for wealthy households falls, which implies that inflation hurts rich households more.

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